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Oscillation theorems for the generalized Liénard system

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ABSTRACT

In this paper we study the qualitative behavior of the solutions of the generalized Liénard system $\dot{x} = h(y - F(x))$ and $\dot{y} = -g(x)$. We present some new conditions under which the solutions of this system are oscillatory. Some examples are used to illustrate our results.

1. Introduction

We consider the following autonomous system of two differential equations

$$\dot{x} = h(y - F(x)), \qquad \dot{y} = -g(x), \tag{1.1}$$

which is a generalized Liénard system.

During the development of radio and vacuum tubes, Liénard equations and the Liénard system were intensely studied as they can be used to model oscillating circuits. Also, the Liénard system has been shown to describe the operation of an optoelectronic circuit that uses a resonant tunneling diode to drive a laser diode to make an optoelectronic voltage controlled oscillator.

The dynamic behaviors of Liénard equations and the Liénard system have been widely investigated due to their application in many fields such as physics, mechanics and engineering technique fields. In such applications, it is important to know the existence of periodic solutions of the Liénard system. In applied science some practical problems associated with the Liénard system can be found in many papers. Hence, it has been the object of intensive analysis by numerous authors (see [1–17] and the references cited therein). On the other hand, many of the other equations and dynamical systems can be transformed to the Liénard system and the existence of periodic solutions plays a key role in characterizing the behavior of a dynamical system. Thus, it is worthwhile investigating the qualitative behavior of the solutions of the Liénard system.

Under the assumptions that the origin is a unique equilibrium for system (1.1), first we study the problem whether all trajectories of this system intersect the vertical isocline $y = F(x)$, which is very important in the global asymptotic stability of the origin, oscillation theory, and existence of periodic solutions. Then we give some oscillation criteria for system (1.1). Our results extend and improve previous results presented in [1–10].

In the following, we shall present the basic assumptions and auxiliary lemmas. We assume that (see [4,11,10])

(C_1) $F(x)$ and $g(x)$ are continuous on $\mathbb{R}$ with $F(0) = 0$, $xg(x) > 0$ for $x \neq 0$ and $h(u)$ is continuous differentiable and strictly increasing with $h(0) = 0$ and $h(\pm\infty) = \pm\infty$,

which guarantee that the origin is the unique critical point of (1.1).

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