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A method to refine the discrete Jensen's inequality for convex and mid-convex functions

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ABSTRACT

A general method is developed to refine the discrete Jensen's inequality in the convex and mid-convex cases. A number of refinements of the discrete Jensen's inequality can be obtained by using the method. The results generalize well known inequalities and give also a new treatment of them. The results are applied to some special situations.

1. Introduction and the main result

The fundamental discrete Jensen's inequalities for convex and mid-convex functions say that

Theorem A (*See [1]*)**.** *Let C be a convex subset of a real vector space X, let $x_i \in C$, and let $p_i \geq 0$ $(i = 1, \ldots, n)$ with $\sum_{i=1}^n p_i = 1$.*
(a) *If $f : C \to \mathbb{R}$ is a convex function, then*

$$f\left(\sum_{i=1}^{n} p_i x_i\right) \leq \sum_{i=1}^{n} p_i f(x_i). \tag{1}$$

(b) *If $f : C \to \mathbb{R}$ is a mid-convex function, and p_i is rational $(i = 1, \ldots, n)$, then (1) also holds.*

Here the function $f : C \to \mathbb{R}$ is called convex if

$$f(\beta x + (1-\beta) y) \leq \beta f(x) + (1-\beta) f(y), \quad x, y \in C, \ 0 \leq \beta \leq 1,$$

and mid-convex if

$$f\left(\frac{x+y}{2}\right) \leq \frac{1}{2} f(x) + \frac{1}{2} f(y), \quad x, y \in C.$$

In the paper [2] we give a refinement of Theorem A, which generalizes and unifies some previous results (see [3,4]). The following conditions are used in [2] and they will be essential in the sequel too.

(H_1) Let V be a real vector space, let C be a convex subset of V, and let $x_1, \ldots, x_n \in C$, where $n \geq 1$ is a fixed integer.
(H_2) Let $p_1, \ldots, p_n > 0$ such that $\sum_{j=1}^n p_j = 1$.
(H_3) Let the function $f : C \to \mathbb{R}$ be convex.
(H_4) Let the function $f : C \to \mathbb{R}$ be mid-convex, and let $p_1, \ldots, p_n$ be rational.

Now we recall the central result of [2].

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