Coupled fixed point theorems for nonlinear contractions in partially ordered *G*-metric spaces

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A B S T R A C T

We prove coupled coincidence and coupled common fixed point theorems for a mixed *g*-monotone mapping satisfying nonlinear contractions in partially ordered *G*-metric spaces. Presented theorems are generalizations of the very recent results of Choudhury and Maity [B.S. Choudhury, P. Maity, Coupled fixed point results in generalized metric spaces, Math. Comput. Modelling 54 (1–2) (2011) 73–79].

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1. Introduction

Many branches of mathematics are concerned with the continuity and convergence of functions. The study of metric spaces allows mathematicians to put this in a more general setting. Many generalizations of the concept of metric space exist in the literature. In 1992, B.C. Dhage introduced a new class of generalized metric space called *D*-metric spaces (see [1,2]). In a subsequent series of papers, Dhage attempted to develop topological structures in such spaces (see [2–4]). He claimed that *D*-metrics provide a generalization of ordinary metric functions and went on to present several fixed point results. In [5], Mustafa in collaboration with B. Sims demonstrated that most of the claims concerning the fundamental topological structure of *D*-metric space are incorrect. They also introduced a valid generalized metric space structure, which they call *G*-metric spaces. Some other papers dealing with *G*-metric spaces are those in [6–10,5,11–14].

On the other hand, there has been recent interest in establishing fixed point theorems in partially ordered complete metric spaces with a contractivity condition which holds for all points that are related by partial ordering. Some of these works are noted in [15–24,14,25–28]. Fixed point problems have also been considered in partially ordered probabilistic metric spaces [20] and in partially ordered *G*-metric spaces [18,14]. In [21], coupled fixed point results in partially ordered metric spaces were established by T. Gnana Bhaskar and V. Lakshmikantham. After the publication of this work, several coupled fixed point and coincidence point results have appeared in recent literatures. Works noted in [29,30,17,18,21,25–28] are some examples of these works.