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Viscosity approximation scheme for a family multivalued mapping

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ABSTRACT

Let K be a nonempty closed convex subset of a real reflexive Banach space X that has weakly sequentially continuous duality mapping J_φ for some gauge φ. Let $T_i : K \to K$ be a family of multivalued nonexpansive mappings with $F := \cap_{i=0}^{\infty} F_{(T_i)} \neq \emptyset$ which is a sunny nonexpansive retract of K with Q a nonexpansive retraction. It is our purpose in this paper to prove the convergence of two viscosity approximation schemes to a common fixed point $\bar{x} = Qf(\bar{x})$ of a family of multivalued nonexpansive mappings in Banach spaces. Moreover, $\bar{x}$ is the unique solution in F to a certain variational inequality.

1. Introduction

Let X be a Banach space with dual X^* and K be a nonempty subset of X. A gauge function is a continuous strictly increasing function $\varphi : R^+ \to R^+$ such that $\varphi(0) = 0$ and $\lim_{t\to\infty} \varphi(t) = \infty$. The duality mapping $J_\varphi : X \to X^*$ associated with a gauge function φ is defined by $J_\varphi(x) := \{f \in X^* : \langle x, f\rangle = \|x\| \|f\|, \|f\| = \varphi(\|x\|)\}, x \in X$, where $\langle ., .\rangle$ denotes the generalized duality pairing. In the particular case $\varphi(t) = t$, the duality map $J = J_\varphi$ is called the normalized duality map. It is noted that $J_\varphi(x) = \frac{\varphi(\|x\|)}{\|x\|} J(x)$, and if X is smooth then J_φ is single valued and norm to weak* continuous (see [1]). When $\{x_n\}$ is a sequence in X, then $x_n \to x (x_n \rightharpoonup x, x_n \rightharpoonup^* x)$ will denote strong (weak, weak*) convergence of the sequence $\{x_n\}$ to x.

Following Browder [2], we say that a Banach space X has weakly continuous duality mapping if there exists a gauge function φ for which the duality map J_φ is single valued and weak to weak* sequentially continuous; i.e. if $\{x_n\}$ is a sequence in X weakly convergent to a point x, then the sequence $\{J_\varphi(x_n)\}$ converges weakly* to $J_\varphi(x)$. It is known that $l_p (1 < p < 1)$ spaces have a weakly continuous duality mapping J_φ with a gauge $\varphi(t) = t^{p-1}$. Setting

$$\Phi(t) = \int_0^{+\infty} \varphi(\tau)\mathrm{d}\tau, \quad t \geq 0,$$

it is easy to see that $\Phi(t)$ is a convex function and that $J_\varphi(x) = \partial\Phi(\|x\|)$, for $x \in X$, where ∂ denotes the subdifferential in the sense of convex analysis. The set K is called proximinal if, for each $x \in X$, there exists an element $y \in K$ such that $\|x - y\| = d(x, K)$, where $d(x, K) = \inf\{\|x - z\| : z \in K\}$. Let $CB(K), C(K), P(K), F(T)$ denote the family of nonempty closed bounded subsets of K, the family of nonempty compact subsets of K, the family of nonempty proximinal bounded subsets of K, and the set of fixed points, respectively. A multivalued mapping $T : K \to CB(K)$ is said to be nonexpansive if

$$H(Tx, Ty) \leq \|x - y\|, \quad x, y \in K,$$

where $H(\cdot, \cdot)$ denotes the Hausdorff metric on $CB(X)$, defined by

$$H(A, B) := \max\left\{\sup_{x\in A}\inf_{y\in B}\|x - y\|, \sup_{y\in B}\inf_{x\in A}\|x - y\|\right\}, \quad A, B \in CB(X).$$

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