



# Intuitionistic fuzzy stability of a Jensen functional equation via fixed point technique

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## ABSTRACT

The object of this paper is to determine Hyers–Ulam–Rassias stability concerning the Jensen functional equation in intuitionistic fuzzy normed space (IFNS) by using the fixed point method. Further, we establish stability of the Cauchy functional equation in IFNS.

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## 1. Introduction and preliminaries

The stability problem of a functional equation was first posed by Ulam [1] concerning the stability of group homomorphism which was answered by Hyers [2] for Banach spaces and then generalized by Aoki [3] and Rassias [4] for additive mappings and linear mappings respectively. Since then several stability problems for various functional equations have been investigated in [5–7]. Recently, fuzzy version is discussed in [8,9]. Quite recently, the stability problem for Jensen functional equation, Pexiderized quadratic functional equation and cubic functional equation is considered in [10–12] respectively in the intuitionistic fuzzy normed spaces; while the idea of intuitionistic fuzzy normed space was introduced in [13] and further studied in [14–17] to deal with some summability problems.

In [18], Radu proposed that the fixed point alternative method is very useful for obtaining the solution of the Ulam problem. The stability problem for the Jensen functional equation was first proved by Kominek [19] and since then several generalizations and applications of this notion have been investigated by various authors, namely Jung [20], Mohiuddine [10], Parnami-Vasudeva [21] and many others. In this paper, we use the fixed point alternative theorem to establish the stability of Hyers–Ulam–Rassias type theorems concerning the Jensen functional equation in intuitionistic fuzzy normed spaces. Also, we determine stability result concerning the Cauchy functional equation in IFNS.

In this section we recall some notations and basic definitions used in this paper.

**Definition 1.1.** A binary operation  $*$  :  $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is said to be a *continuous  $t$ -norm* if it satisfies the following conditions:

(a)  $*$  is associative and commutative, (b)  $*$  is continuous, (c)  $a * 1 = a$  for all  $a \in [0, 1]$ , (d)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for each  $a, b, c, d \in [0, 1]$ .

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