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Finite iterative algorithms for extended Sylvester-conjugate matrix equations

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ABSTRACT

An iterative algorithm is presented for solving the extended Sylvester-conjugate matrix equation. By this iterative method, the solvability of the matrix equation can be determined automatically. When the matrix equation is consistent, a solution can be obtained within finite iteration steps for any initial values in the absence of round-off errors. The algorithm is also generalized to solve a more general complex matrix equation. Two numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

For complex matrices, besides similarity there is another important equivalence relation, consimilarity [1–3]. Two complex matrices A and B are said to be consimilar if there exists an invertible complex matrix P such that $\overline{P}^{-1}AP = B$, where $\overline{P}$ denotes the matrix obtained by taking the complex conjugate of each element of P. In [1,4,5], it was shown that the consistency of the matrix equation $AX - \overline{X}B = C$ can be characterized by the consimilarity of two matrices. Very recently, in [6] some explicit expressions of the solution to the matrix equation $AX - \overline{X}B = C$ have been established by means of a real representation of a complex matrix proposed in [7], and it was shown that there exists a unique solution if and only if $A\overline{A}$ and $B\overline{B}$ have no common eigenvalues. In addition, the matrix equation $X - A\overline{X}B = C$ was investigated in [7] with the aid of a real representation of a complex matrix, the consistency and solutions of this equation were established in terms of its real representation matrix equation. Recently, based on Smith form reduction of polynomial matrices, explicit solutions have been given in [8] for a class of complex matrix equations, the so-called Yakubovich-conjugate matrix equations. Very recently, a general class of Sylvester-polynomial-conjugate matrix equations, which include the above-mentioned matrix equations as its special cases, has been investigated in [9]. The complete solution to the Sylvester-polynomial-conjugate matrix equation was obtained in terms of the Sylvester-conjugate sum, and such a proposed solution can provide all the degrees of freedom with an arbitrarily chosen parameter matrix. The main tool in [9] was the so-called conjugate product of complex polynomial matrices proposed in [10]. However, the methods in the aforementioned literature are not suitable for the matrix equation $AXB + C\overline{X}D = F$, which will be studied in this paper. For convenience, this equation will be referred to as the extended Sylvester-conjugate matrix equation in what follows. Obviously, the extended Sylvester-conjugate matrix equations include the matrix equations $AX - \overline{X}B = C$ and $X - A\overline{X}B = C$ as special cases. Another motivation of investigating

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