Mathematical and Computer Modelling 54 (2011) 2355–2362

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

A new comprehensive class of analytic functions defined by multiplier transformation

Alina Alb Lupaş *

Department of Mathematics and Computer Science, University of Oradea, str. Universitatii nr. 1, 410087 Oradea, Romania

ARTICLE INFO

Article history:
Received 21 March 2011
Received in revised form 20 May 2011
Accepted 20 May 2011

Keywords:
Differential subordination
Convex function
Best dominant
Differential operator

ABSTRACT

For functions belonging to the class $SI_m(\delta)$, $\delta \in [0, 1)$ and $m \in \mathbb{N}$, of normalized analytic functions in the open unit disc U, which is investigated in this paper, the author derives several interesting differential subordination results. These subordinations are established by means of a special case of the multiplier transformations $I(m, \lambda, l) f(z)$ namely

$$I(m, \lambda, l) f(z) = z + \sum_{j=n+1}^{\infty} \left(\frac{\lambda (j-1) + l + 1}{l+1} \right)^m a_j z^j,$$

where $n \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$, $\lambda, l \geq 0$ and $f \in \mathcal{A}_n$,

$$\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1} z^{n+1} + \cdots,\ z \in U\}.$$

A number of interesting consequences of some of these subordination results are discussed. Relevant connections of some of the new results obtained in this paper with those in earlier works are also provided.

1. Introduction

Denote by U the unit disc of the complex plane, $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}(U)$ the space of holomorphic functions in U.

Let $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1} z^{n+1} + \cdots,\ z \in U\}$ and $\mathcal{H}[a, n] = \{f \in \mathcal{H}(U) : f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots,\ z \in U\}$ for $a, a_n, a_{n+1}, \ldots \in \mathbb{C}$ and $n \in \mathbb{N}$.

Definition 1. For $f \in \mathcal{A}_n$, $n \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$, $\lambda, l \geq 0$, the operator $I(m, \lambda, l) f(z)$ is defined by the following infinite series

$$I(m, \lambda, l) f(z) = z + \sum_{j=n+1}^{\infty} \left(\frac{\lambda (j-1) + l + 1}{l+1} \right)^m a_j z^j.$$

Remark 1. It follows from the above definition that

$$I(0, \lambda, l) f(z) = f(z),$$
$$(l+1) I(m+1, \lambda, l) f(z) = (l+1-\lambda) I(m, \lambda, l) f(z) + \lambda z (I(m, \lambda, l) f(z))', \quad z \in U.$$

* Tel.: +40 74475537.
E-mail address: dalb@uoradea.ro.

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.05.044