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Common fixed points in cone metric spaces for *CJM*-pairs

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ABSTRACT

In this paper we introduce some contractive conditions of Meir–Keeler type for two mappings, called f-*MK*-pair mappings and f-*CJM*-pair (from Ciric, Jachymski, and Matkowski) mappings, in the framework of regular cone metric spaces and we prove theorems which guarantee the existence and uniqueness of common fixed points. We give also a fixed point result for a multivalued mapping that satisfies a contractive condition of Meir–Keeler type. These results extend and generalize some recent results from the literature. To conclude the paper, we extend our main result to non-regular cone metric spaces by using the scalarization method of Du.

1. Introduction and preliminaries

Fixed point theory is an important current topic in nonlinear analysis. For the most important contributions as regards the metric and non-metric settings, see [1,2] (and the references therein) and [3]. Let B be an ordered linear space with a cone K and a class of convergent sequences in B. For more details on convergence structures see [4,5].

Definition 1. Let X be a nonempty set. Suppose that the mapping $d : X \times X \to B$ satisfies:

(i) $0 \leqslant d(x, y)$ for all $x, y \in X$, and $d(x, y) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(iii) $d(x, y) \leqslant d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a *cone metric (also a K-metric)* on X, and (X, d) is called a *cone metric space (also a K-metric space)*.

A sequence $\{x_n\} \subset X$ is called convergent if there exists an element $x \in X$ such that the sequence $\{d(x_n, x)\}$ is convergent to zero in the space B. Zabrejko [4] presented a very interesting revised version of the fixed point theory in K-metric and K-normed linear spaces and gave three fixed point theorems that cover numerous applications, e.g. in numerical methods and the theory of integral equations (see also [6]). Clearly one can formulate specializations of these results for special types of K-metric. For more considerations relating to K-metric spaces, in terms of the weakly Picard operators, the reader can see [7]. In [7] the authors have also considered K-metrics induced by a functional.

Huang and Zhang [8] generalized the concept of a metric space, replacing the set of real numbers by an ordered Banach space, and obtained some fixed point theorems for mappings satisfying different contractive conditions. Vetro [9], in the framework of cone metric spaces, introduced a generalized contractive condition and proved a common fixed point theorem for a pair of weakly compatible mappings. This theorem generalizes some results of Huang and Zhang [8]. Further to this, Abbas and Jungck [10] proved common fixed point theorems for a pair of weakly compatible mappings. Recently, Rezapour and Hamlbarani [11], omitting the assumption of normality, obtained generalizations of some results of [8]. Following these

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