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Asymptotic properties and simulations of a stochastic logistic model under regime switching

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ABSTRACT

Taking both white noise and colored environmental noise into account, a general stochastic logistic model under regime switching is proposed and studied. Sufficient conditions for extinction, nonpersistence in the mean, weak persistence, stochastic permanence and global attractivity are established. The critical number between weak persistence and extinction is obtained. Moreover, some simulation figures are introduced to illustrate the main results.

1. Introduction

Logistic system is one of the most classical models in both ecology and mathematical ecology owing to its theoretical and practical significance. The classic deterministic logistic equation is usually denoted by:

$$\mathrm{d}x(t)/\mathrm{d}t = x(t)[r - ax(t)] \tag{1}$$

for $t \geq 0$ with initial value $x(0) = x_0 > 0$, $x(t)$ is the population size at time t. r stands for the growth rate and a denotes the intraspecific competition coefficient, i.e., r/a is the carrying capacity. We refer the reader to May [1] for a detailed model construction.

Owing to its theoretical and practical significance, model (1) and its various generalization forms have been extensively investigated and many important results on the global dynamics of solutions have been found; see e.g. Golpalsamy [2], Kuang [3], Faria [4], Li and Chen [5], Li et al. [6] and the references therein. Particularly, the books by Golpalsamy [2] and Kuang [3] are good references in this area.

On the other hand, in the real world, population system is inevitably affected by the environmental noise which is an important component in an ecosystem (see e.g. [7–10]). The deterministic systems assume that parameters in the models are all deterministic irrespective of environmental fluctuations. Hence, they have some limitations in mathematical modeling of ecological systems, besides they are quite difficult to fitting data perfectly and to predict the future dynamics of the system accurately [11]. May [1] pointed out the fact that due to environmental noise, the birth rate, carrying capacity, competition coefficient and other parameters involved in the system exhibit random fluctuation to a greater or lesser extent.

Generally speaking, there are two types of environmental noise. One is colored noise, another is white noise. First of all, let us consider a classical colored noise, say telegraph noise. The telegraph noise can be illustrated as a switching

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