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Fuzzy $*$-homomorphisms and fuzzy $*$-derivations in induced fuzzy C^*-algebras

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ABSTRACT

Using the fixed point method, we prove the Hyers–Ulam stability of the Cauchy–Jensen functional equation and of the Cauchy–Jensen functional inequality in fuzzy Banach $*$-algebras and in induced fuzzy C^*-algebras.

Furthermore, using the fixed point method, we prove the Hyers–Ulam stability of fuzzy $*$-derivations in fuzzy Banach $*$-algebras and in induced fuzzy C^*-algebras.

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1. Introduction and preliminaries

The theory of fuzzy space has progressed greatly, developing the theory of randomness. Some mathematicians have defined fuzzy norms on a vector space from various points of view [1–6]. Following Cheng and Mordeson [7], Bag and Samanta [1] gave an idea of fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [8] and investigated some properties of fuzzy normed spaces [9].

We use the definition of fuzzy normed spaces given in [1,5,10] to investigate a fuzzy version of the Hyers–Ulam stability for the Cauchy–Jensen functional equation in the fuzzy normed $*$-algebra setting.

Definition 1.1 (*[1,5,10,11]*). Let X be a complex vector space. A function $N : X \times \mathbb{R} \to [0, 1]$ is called a *fuzzy norm* on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$,

(N_1) $N(x, t) = 0$ for $t \leq 0$;
(N_2) $x = 0$ if and only if $N(x, t) = 1$ for all $t > 0$;
(N_3) $N(cx, t) = N(x, \frac{t}{|c|})$ if $c \in \mathbb{C} \setminus \{0\}$;
(N_4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$;
(N_5) $N(x, \cdot)$ is a non-decreasing function of $\mathbb{R}$ and $\lim_{t\to\infty} N(x, t) = 1$;
(N_6) for $x \neq 0$, $N(x, \cdot)$ is continuous on $\mathbb{R}$.

The pair (X, N) is called a *fuzzy normed vector space*.

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