



The stochastic stability and bifurcation behavior of an Internet congestion control model

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ABSTRACT

A stochastic differential equation modeling an Internet congestion control model is investigated. By analyzing the Lyapunov exponent, invariant measure and singular boundary theory, some new criteria ensuring stochastic stability and bifurcation are obtained. Numerical simulation results are given to support the theoretical predictions.

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1. Introduction

Internet congestion control is an algorithm to allocate available resources to competing sources efficiently, so as to avoid congestion collapse. The whole Internet congestion and avoidance mechanism is a combination of the end-to-end TCP congestion control mechanism [1] at the end hosts and the queue management mechanism at the routers. The basis of TCP congestion control lies in the Additive Increase Multiplicative Decrease (AIMD) mechanism that halves the congestion window for every window containing a packet loss, and increases the congestion window by roughly one segment per Round Trip Time (RTT) otherwise [2]. The queue management mechanism is meant to control the congestion level at each router through different kinds of Active Queue Management (AQM) mechanisms, e.g. Drop Tail [1], Random Early Detection (RED) [3], Random Early Marking (REM) [4], Virtual Queue (VQ) [5], and Adaptive Virtual Queue (AVQ) [6]. Understanding the dynamics and stability of the congestion control algorithm in the Internet has been the focus of intense research in the last few years [1–11].

In [10], the Internet congestion control model is described by

$$\begin{cases} \dot{W}(t) = \frac{1}{R(t)} - \frac{W(t) \times W(t - R(t))}{2R(t - R(t))} p(t - R(t)), \\ \dot{q}(t) = N(t) \frac{W(t)}{R(t)} - C, \end{cases} \quad (1.1)$$

where $W(t)$ denotes the average of TCP windows size (packets), $q(t)$ is the average of queue length (packets), $p(\cdot)$ is the probability function of a packet mark, $N(t)$ is the number of TCP sessions, C is the queue capacity (packets/s) and $R(t)$ is the Round Trip Time(s) which consists of the propagation delay and queuing delay. When the queuing delay is much smaller than

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