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# On the mathematical theory of living systems, I: Complexity analysis and representation

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ABSTRACT

This paper is the first one of a sequel devoted to the challenging goal of developing a mathematical theory for living systems. We consider systems constituted of a number of living entities, called active particles, which have the ability to express specific strategies and interact with other entities. The author proposes a personal path, starting from the identification of a number of common features of living systems that can be viewed as sources of complexity, focusing specifically on the representation of systems based also on a strategy to reduce their complexity. The overall system is decomposed into functional subsystems whose representation is delivered by a probability distribution over the microscopic state of the active particles belonging to such system. Looking ahead, this paper indicates some guidelines to derive mathematical structures, where interactions involving active particles are nonlinearly additive.



## 1. Introduction

The study of complex systems, namely systems of many individuals interacting in a nonlinear manner, has received in recent years a remarkable increase of interest from applied mathematicians [1]. Their interest follows that, already well settled, of physicists and researchers in various other fields as engineering, economy or social sciences.

Focusing on large systems constituted by several living entities, it is very difficult to understand and model them based on the sole description of the dynamics and interactions of a few individual entities localized in space and time. In other words, the modeling of individual dynamics does not lead in a straightforward way to a mathematical description of collective emerging behaviors. Understanding the rôle of nonlinear interactions is indeed one of the greatest challenges in the study of complex systems, considering that they are at the core of the emergence of qualitatively different states, namely, new collective states that are not mere combinations of the states of the individual units belonging to the system. In other words, the dynamics of each entity is not determined by the dynamics of all other entities, but by their action as a whole.

A remarkable conceptual difficulty arising from dealing with living system is the lack of fundamental paradigms (first principles) about equilibrium and conservation rules. Some reasonings on this matter are offered in various papers [2–8], and books [9–11]. This lack of paradigms has prevented that various existing models of biological phenomena, even successful under many respects, could generate a biological–mathematical theory analogous to historical theories born from the encounter between mathematical and physical sciences.

Moreover, causality principles are almost always lost due to the heterogeneously distributed ability of living systems to express a strategy often related to their will to survive. More precisely, it is a complex output related to the afore-mentioned ability to express specific strategies that may change not only due to the presence of other entities, but also as a consequence of environmental conditions.

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