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Troesch's problem: A B-spline collocation approach

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ABSTRACT

A finite-element approach, based on cubic B-spline collocation, is presented for the numerical solution of Troesch's problem. The method is used on both a uniform mesh and a piecewise-uniform Shishkin mesh, depending on the magnitude of the eigenvalues. This is due to the existence of a boundary layer at the right endpoint of the domain for relatively large eigenvalues. The problem is also solved using an adaptive spline collocation approach over a non-uniform mesh via exploiting an iterative scheme arising from Newton's method.

The convergence analysis is discussed and is shown to depend on the eigenvalues; in particular, the rate of convergence is calculated using the double-mesh principle. To demonstrate the efficiency of the method, a number of special cases are considered. The numerical solutions are compared with both the analytical solutions and other existing numerical solutions in the literature. It is observed that the results obtained by this method are quite satisfactory and accurate, and the method is applicable for a wide range of cases when contrasted with other available solutions.

1. Introduction

In this paper, we will manipulate B-spline functions to develop a collocation method for solving the nonlinear Troesch problem. This problem arises in the investigation of the confinement of a plasma column by radiation pressure, which was described and solved by Weibel [1]. The analytical solution of this problem was obtained by Roberts and Shipman [2]; the closed-form solution was expressed in terms of the Jacobi elliptic function.

A number of approaches have been used to obtain a numerical solution to this problem, such as the shooting method and a decomposition scheme representing the nonlinear problem as an integral equation. Some references for such numerical solutions can be found in [3,4]. In particular, Scott [5] used an invariant imbedding method to solve Troesch's problem, while Khuri [4] used a numerical method based on Laplace transformation and a modified decomposition technique to obtain an approximate solution of the same problem. Feng et al. [3] solved this problem numerically using a modified homotopy perturbation technique. Chang and Chang [6] developed a new technique for calculating the one-dimensional differential transform of nonlinear functions; the algorithm was illustrated by studying several nonlinear ordinary differential equations, including Troesch's problem. Chang [7] proposed a new algorithm based on the variational method and variable transformation to solve Troesch's problem.

The cubic B-spline finite-element method (see [8–10]) is often used for solving nonlinear problems [11] that arise in engineering applications. In this paper, cubic B-spline functions are utilized to develop a collocation method for solving Troesch's problem. A uniform mesh is used when the eigenvalues are relatively small, $0 < \lambda \leq 5$; however, for large eigenvalues we will not be able to get uniform convergence at all the mesh points because of the existence of a boundary

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