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Local and global bifurcations in a small power system

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ABSTRACT

This paper presents a number of local and one global (homoclinic loop) bifurcation observed in a small power system involving a load tap changer transformer and an induction motor. Besides the well-known generic local bifurcations (saddle-node and generic Hopf), the paper presents examples of higher codimension bifurcations, such as fold with double zero eigenvalues, degenerate Hopf, and swallowtail bifurcation.

1. Introduction

Bifurcation theory is a powerful analysis tool in the research area of dynamical systems. Basic references, like [1], provide the general background and the taxonomy of bifurcations or equivalently catastrophes [2]. In the field of power system engineering several early research papers have dealt with the analysis of bifurcations. An overview of early results has been given in [3]. Generic Saddle Node bifurcation and its geometric and algebraic properties with respect to system loading margins have been discussed in [4,5] with recent applications in [6] and applied in medium and large scale power systems [7]. Hopf bifurcations are frequently met in power systems, since the interaction of the dynamics of power system components may lead to unstable operation through emergence of oscillations, as has been analyzed in [8,9]. Unstable oscillations eventually impose restrictions on the available transfer capability of power systems [10], which may also affect electricity markets with bilateral or multilateral transactions, as shown in [11]. Furthermore, the concept of special (degenerate) cases of Hopf bifurcation is illustrated in [12,13] from different points of view, while the nature of global bifurcation has been discussed in [14]. Also a more complex (chaotic) behavior of power systems has been studied in [15], based however on models that are not so much used in recent years.

The main goal of this paper is to present a variety of bifurcations, of co-dimensions ranging from 1 to 3, that can be experienced during the operation of a small power system first introduced in [16], using standard models of a Load Tap Changer and an Induction Motor. A similar system has been analyzed for bifurcations in [17]. Though the system is small, it can be considered representative of a radial link feeding a load with significant motor component and it can serve for studying bifurcation phenomena in power systems without resorting to artificial mathematical models.

In this paper, bifurcations are studied by focusing on state-space analysis. Besides generic bifurcations, system behavior is studied in the presence of higher co-dimension bifurcations, such as double-zero fold, degenerate Hopf, and swallowtail bifurcation. The paper aims to show what is a reasonable succession of these bifurcations in a system, and which of these phenomena are likely to be observed in real-life systems.

2. Bifurcation theory overview

2.1. Definitions and classification

We consider the dynamical system described by ODEs of the following form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{p}) \tag{1}$$

where $\mathbf{x}$ is the $n \times 1$ vector of state variables and $\mathbf{p}$ is the $k \times 1$ vector of parameters, while $\mathbf{f}$ is a vector of n *smooth* functions. It is well known [1,2,18] that the term *bifurcation* is used to indicate a radical change in the qualitative structure of a nonlinear system following a

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