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Notes on "Application of the Hamiltonian approach to nonlinear oscillators with rational and irrational elastic terms"

1. Introduction

Very recently, Yildirim et al. [1] applied the so-called Hamiltonian approach (HA) to obtain analytical approximate solutions for three well-known nonlinear oscillators. The authors mentioned that this approach is a kind of energy method with a vast application in conservative oscillatory systems, and they applied the approach to nonlinear oscillators with rational and irrational elastic terms. They also pointed out that a comparison of the approximate solutions and the exact ones proves that the HA is quite accurate in nonlinear analysis of dynamical systems. Their results are based on a new method that has been developed [2] which can be applied to conservative nonlinear oscillators with odd elastic terms. In this paper, we will demonstrate that, when the trial function $u(t) = A\cos\omega t$ is used in the HA, the results obtained are the same as those one can obtain using the known first-order harmonic balance method (HBM), and that the HA can be derived from the equations obtained when the first-order HBM is considered. Therefore, the application of the HA in [1] could be considered as a corollary of the first-order HBM, and all the results obtained are the same as those obtained by applying the HBM. Finally, we include additional comments about the analytical approximate expressions for the frequency given in [1] as well as a general expression for this frequency for an extensive set of conservative nonlinear oscillators.

2. Derivation of the HA equations in [1] from the first-order HBM

Consider the simplest nonlinear conservative autonomous system encountered in the theory of oscillations with one degree of freedom, whose motion is governed by the following dimensionless second-order differential equation:

$$\frac{d^2u}{dt^2} + f(u) = 0, \qquad u(0) = A, \qquad \frac{du}{dt}(0) = 0, \tag{1}$$

where the nonlinear restoring-force function $f(u)$ is odd, i.e. $f(-u) = -f(u)$, and satisfies $uf(u) > 0$ for $u \in [-A, A]$, $u \neq 0$ [3]. It is obvious that $u = 0$ is the equilibrium position. This condition is not considered in [1], but it is necessary to apply Eq. (4) presented in Yildirim et al.'s paper. The motion is assumed to be periodic, and the problem is to determine the angular frequency of oscillation ω and the corresponding solution $u(t)$ as functions of the system parameters and the amplitude A.

The HBM provides a general technique for obtaining analytical approximate expressions for the frequency and the periodic solution of nonlinear oscillators by using a truncated Fourier series representation [4,5]. To solve Eq. (1) by the HBM, a new independent variable $\tau = \omega t$ is introduced, so Eq. (1) can be rewritten as

$$\omega^2\frac{d^2u}{d\tau^2} + f(u) = 0, \qquad u(0) = A, \qquad \frac{du}{d\tau}(0) = 0. \tag{2}$$

The new variable is chosen in such a way that the solution of Eq. (2) is a periodic function of τ of period 2π [3]. Since the restoring force $f(u)$ is an odd function of u, the periodic solution $u(\tau)$ has a Fourier series representation which contains only odd multiples of τ, and the first-order harmonic balance solution takes the form

$$u(\tau) = A\cos\tau. \tag{3}$$

Observe that $u(\tau)$ satisfies the initial conditions, Eq. (2), and it is the trial function used in [1]. Substituting Eq. (3) into Eq. (2) gives

$$-\omega^2 A\cos\tau + f(A\cos\tau) = 0. \tag{4}$$

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