



# Critical investigation of preconditioned GMRES via incomplete LU factorization applied to power flow simulation

José Eduardo O. Pessanha<sup>a,\*</sup>, Carlos Portugal<sup>b</sup>, Ricardo Prada<sup>b</sup>, Alex R. Paz<sup>b</sup>

<sup>a</sup> Power Quality Laboratory, Institute of Electrical Engineering, UFMA, Campus do Bacanga, São Luís, Ma 65080-040, Brazil

<sup>b</sup> PUC-Rio-DEE, Department of Electrical Engineering, Rua Marquês de São Vicente, 225, Gávea, RJ 22453-900, Brazil

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## ABSTRACT

For solving the power flow sublinear problem efficiently by the GMRES preconditioned via incomplete LU factorization (ILU), this paper investigates causes associated to the preconditioner low quality and proposes a method to improve it and the GMRES convergence rate as well. The goal is provide a well-organized ILU-GMRES for solving linear systems of difficult solution comprising ill-conditioned coefficient matrices, normally associated to heavy load power systems. The investigations reveal that a dropping rule for nonzero elements (fill-ins) based on a relative tolerance may introduce large errors during the preconditioner construction, lowering its quality and the GMRES performance. Based on that, it is proposed a fill-in dropping rule making use of two criteria; one based on the resulting error and the other based on a relative tolerance, applied to the preconditioner lower (L) and upper (U) triangular matrices, respectively. Ordering schemes are also considered. Numerical experiments taking into account different power system configurations operating under heavy load conditions corroborate the efficiency of such strategies.

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## 1. Introduction

In order to supply and reach more costumers with a reliable and continuous service, power systems around the world are continuously increasing in terms of dimension, complexity and load level. As a result, computer simulations are facing large and complex mathematical systems, such as those found in the solution of large sparse linear systems of type (1), frequently the most time-consuming part of the computation, demanding not only for high quality computer technologies but also for robust numerical methods. In general, these systems are solved through direct methods and despite of their robustness, they tend to have need of a predictable amount of resources in terms of computational time and storage [1,2], or even fail for ill-conditioned and/or very large coefficient matrices (A) [3]. Preconditioned Krylov subspace iterative methods have proven to be suitable alternatives for these cases; in general, their requirements for memory storage are usually smaller by orders of magnitude [4] and present relative simplicity of implementation in parallel algorithms, conversely for direct methods [5–7].

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \quad (1)$$

One of the first applications of preconditioned iterative methods in power system problems was presented by Decker et al. [8]. The authors combined the LU factorization method with the Conjugate Gradient in transient stability analysis using parallel computation. In load flow analysis, Ref. [9] applied the Preconditioned Conjugate Gradient in dc and fast decoupled models. However, as iterative methods and preconditioned techniques improved, new contributions came up, mainly related to the preconditioned Generalized Minimum Residual method – GMRES [10] and variants, such as GMRES(m) and GMRES-E [11]. The GMRES performance has been investigated in load flow applications [12,13], state estimation, power system security and transient stability [14]. Ref. [15] proposes an adaptive preconditioner constructed for Jacobian-free Newton-GMRES(m) method for solving coordination equations in distributed simulations of power systems. An extension of this work is presented in [16] where a new algorithm for distributed transient stability simulation of interconnected power systems based on a Jacobian-free Newton-GMRES(m) is proposed. Finally, Ref. [17] proposes a method comprising continued power flow and GMRES for voltage collapse analysis referred to as CPF-GMRES method.

The present work is focused on the iterative solution of the power flow sublinear problem (Appendix A) of type (1) using a well-organized preconditioned GMRES method (Appendix B), with the preconditioning matrix constructed by means of incomplete LU factorization based on the power system Jacobian matrix calculated at the very first Newton–Raphson iteration. Normally, if the power system is heavy loaded, the condition number of the Jacobian matrix is very high resulting in a low quality ILU

\* Corresponding author. Address: Avenida dos Portugueses s/n, Campus do Bacanga, São Luís, Ma 65080-040, Brazil. Tel.: +55 98 3301 9204.

E-mail addresses: [pessanha@dee.ufma.br](mailto:pessanha@dee.ufma.br) (José Eduardo O. Pessanha), [portugal@ele.puc-rio.br](mailto:portugal@ele.puc-rio.br) (C. Portugal), [prada@ele.puc-rio.br](mailto:prada@ele.puc-rio.br) (R. Prada), [alarpazp@gmail.com](mailto:alarpazp@gmail.com) (A.R. Paz).