Mathematical and Computer Modelling 54 (2011) 3098–3115

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Fractional representation formulae and right fractional inequalities

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ARTICLE INFO

Article history:
Received 11 June 2011
Accepted 29 July 2011

Keywords:
Fractional derivatives
Fractional representations
Fractional inequalities

ABSTRACT

Here we prove fractional representation formulae involving generalized fractional derivatives, Caputo fractional derivatives and Riemann–Liouville fractional derivatives. Then we establish Poincaré, Sobolev, Hilbert–Pachpatte and Opial type fractional inequalities, involving the right versions of the abovementioned fractional derivatives.

1. Introduction

This article continues the author's monograph [1], and articles [2–5].

Among others, we establish fractional representation formulae for left and right generalized fractional derivatives, left and right Caputo fractional derivatives, and right Riemann–Liouville fractional derivatives.

Then, based on these formulae, we prove new fractional differentiation inequalities of Poincaré type, Sobolev type, Hilbert–Pachpatte type, and Opial type.

These inequalities involve all right fractional derivatives mentioned above.

To give a flavour of our work, we mention the following.

Let $f \in AC^m([a,b])$ (means $f^{(m-1)}$ is absolutely continuous), $m \in \mathbb{N}$, $m := \lceil \alpha \rceil$ (ceiling of the number), $\alpha \notin \mathbb{N}$, $\alpha \geq \gamma + 1$, $\gamma \geq 0$, $n := \lceil \gamma \rceil$. Assume $f^{(k)}(b) = 0$, $k = n, n+1, \ldots, m-1$, $\overline{D}_{b-}^{\alpha} f \in L_\infty(a,b)$ (where $\overline{D}_{b-}^{\alpha} f$ is the right Caputo fractional derivative); see Theorem 15.

Then we have the representation formula

$$\overline{D}_{b-}^{\gamma} f(x) = \frac{1}{\Gamma(\alpha-\gamma)} \int_x^b (t-x)^{\alpha-\gamma-1} \left(\overline{D}_{b-}^{\alpha} f\right)(t)\,dt, \tag{1}$$

$\forall x \in [a,b]$, Γ stands for gamma function.

Using (1) and $p, q > 1 : \frac{1}{p} + \frac{1}{q} = 1$, we prove the following Poincaré type right Caputo fractional inequality; see Theorem 21,

$$\int_a^b \left|\overline{D}_{b-}^{\gamma} f(x)\right|^q dx \leq \frac{(b-a)^{q(\alpha-\gamma)}}{\left\{(\Gamma(\alpha-\gamma))^q (p(\alpha-\gamma-1)+1)^{(q/p)} q(\alpha-\gamma)\right\}} \cdot \left(\int_a^b \left|\overline{D}_{b-}^{\alpha} f(\zeta)\right|^q d\zeta\right). \tag{2}$$

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doi:10.1016/j.mcm.2011.07.040