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Strong convergence of shrinking projection methods for a family of pseudocontractive mappings in Hilbert spaces

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ABSTRACT

The shrinking projection method has gained more and more attention as a powerful tool for the approximation of a fixed point of nonlinear mappings. In this paper, we introduce a new shrinking projection method for the approximation of fixed points of a family of pseudocontractive mappings in a Hilbert space. Using this method, we also deal with the problem of finding a common zero of a family of monotone operators and obtain a strong convergence theorem.

1. Introduction

Let H be a real Hilbert space and C be a nonempty closed convex subset of H. Let T be a self-mapping of C. We use $F(T)$ to denote the set of fixed points of T (i.e., $F(T) = \{x \in C : Tx = x\}$).

Definition 1.1 (*[1]*). A mapping $T : C \to C$ is said to be strict pseudo-contraction if there exists a constant $0 \le k < 1$ such that

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + k\|(I - T)x - (I - T)y\|^2, \tag{1.1}$$

for all $x, y \in C$. If $k = 1$, then T is said to be pseudo-contraction, i.e.,

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + \|(I - T)x - (I - T)y\|^2, \tag{1.2}$$

equivalent,

$$\langle (I - T)x - (I - T)y, x - y\rangle \ge 0, \tag{1.3}$$

for all $x, y \in C$.

A mapping T is said to be nonexpansive, if $\|Tx - Ty\| \le \|x - y\|$, for all $x, y \in C$.

It is obvious that all nonexpansive mappings and strictly pseudocontractive mappings are pseudocontractive mappings.

Iterative methods for finding fixed points of nonexpansive mappings are an important topic in the theory of nonexpansive mappings and have wide applications in a number of applied areas, such as the convex feasibility problem [2–4], the split feasibility problem [5–7] and image denoising and deblurring [8–10]. However, the Picard sequence $\{T^n x\}_{n=0}^{\infty}$ often fails to converge even in the weak topology. Thus averaged iterations prevail. Mann's iteration is one of the type and is defined by:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n, \quad n \ge 0, \tag{1.4}$$

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