



On the system of two difference equations of exponential form:

$$x_{n+1} = a + bx_{n-1}e^{-y_n}, \quad y_{n+1} = c + dy_{n-1}e^{-x_n}$$

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ABSTRACT

It is the goal of this paper to study the boundedness, the persistence and the asymptotic behavior of the positive solutions of the system of two difference equations of exponential form

$$x_{n+1} = a + bx_{n-1}e^{-y_n}, \quad y_{n+1} = c + dy_{n-1}e^{-x_n},$$

where a, b, c, d are positive constants, and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive real values.

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1. Introduction

In [1], the authors studied the boundedness, the asymptotic behavior, the periodic character of the solutions and the stability character of the positive equilibrium of the difference equation

$$x_{n+1} = a + bx_{n-1}e^{-x_n},$$

where a, b are positive constants and the initial values x_{-1}, x_0 are positive numbers. Furthermore, in [1] the authors used a as the immigration rate and b as the growth rate in the population model. In fact, this was a model suggested by the people from the Harvard School of Public Health; studying the population dynamics of one species x_n .

Motivated by the above paper we will extend the above difference equation to a system of difference equations; our goal will be to investigate the boundedness, the persistence and the asymptotic behavior of the positive solutions of the system of two difference equations of exponential form

$$\begin{aligned} x_{n+1} &= a + bx_{n-1}e^{-y_n}, \\ y_{n+1} &= c + dy_{n-1}e^{-x_n} \end{aligned} \quad (1.1)$$

where a, b, c, d are positive constants and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive real values; also, b will be growth rate of species x_n and d will be the growth rate of species y_n ; in addition, a will be the immigration rate into species x_n and c will be the immigration rate into species y_n . Furthermore, system (1.1) can be applied as a two directional interactive and invasive species model where species x_n and y_n affect each other's population in both directions. Observe that it is very crucial for every positive solution of system (1.1) to be bounded as the population of species x_n and y_n cannot grow infinitely large due to the limited resources. Moreover, the equilibrium point $(\bar{x}, \bar{y})$ of system (1.1) is considered to be the natural ideal

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