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Existence of multiple periodic solutions for first order functional differential equations

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ABSTRACT

We obtain sufficient conditions for the existence of three T-periodic solutions of the first order functional differential equation

$$u'(t) = a(t)g(u(t))u(t) - b(t)f(u(t - \tau(t))),$$

where $a, b, \tau \in C(\mathbb{R}, \mathbb{R})$ are T-periodic functions, $f, g \in C(\mathbb{R}, \mathbb{R})$, and g is not necessarily bounded. As an application of our theorem, we also derived criteria for the existence of three T-periodic solutions of the eigenvalue problem

$$u'(t) = a(t)g(u(t))u(t) - \lambda b(t)f(u(t - \tau(t))),$$

where λ is a positive parameter. Our analysis mainly relies on the lower and upper solution method and the topological degree theory. Examples are provided to apply our results.

1. Introduction

Functional differential equations with periodic delays appear in a number of applications such as in species growth models, the production of blood cells, the control of testosterone levels in the blood stream, and so on. The reader may refer to [1–4] for such applications. In this paper, we are concerned with the existence of T-periodic solutions of the first order functional differential equation

$$u'(t) = a(t)g(u(t))u(t) - b(t)f(u(t - \tau(t))), \tag{1.1}$$

where $a, b, \tau \in C(\mathbb{R}, \mathbb{R})$ are T-periodic functions and $f, g \in C(\mathbb{R}, \mathbb{R})$. When $g(x)$ is constant or bounded, the existence of T-periodic solutions of (1.1) or its variations has been extensively investigated in the literature, for example, in [5–11]. When $g(x)$ is not necessarily bounded, papers [12,13] studied a variation of (1.1), namely,

$$u'(t) = a(t)g(u(t))u(t) - \lambda b(t)f(u(t - \tau(t))), \tag{1.2}$$

where λ is a positive parameter, and obtained sufficient conditions for the existence of one and two positive T-periodic solutions when λ is in different intervals. The main tool in [12,13] is the fixed point index theory and no result is given there for the existence of three T-periodic solutions.

In this paper, we will apply the lower and upper solution method and the Leray–Schauder degree theory to study the existence of three T-periodic solutions of (1.1). As is well known, the lower and upper solution method and the Leray–Schauder degree theory have been very useful in the investigation of existence of solutions of boundary value problems for differential equations. We refer the reader to [14,15] for some related work. In this paper, under the condition

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