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On the brachistochrone of a variable mass particle in general force fields

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ABSTRACT

The problem of the brachistochronic motion of a variable mass particle is considered. The particle moves through a resistant medium in the field of arbitrary active forces. Beginning from these general assumptions, and applying Pontryagin's minimum principle along with singular optimal control theory, a corresponding two-point boundary value problem is obtained and solved. The solution proposed involves an appropriate numerical procedure based upon the shooting method. In this numerical procedure, the evaluation of ranges for unknown values of costate variables is avoided by the choice of a corresponding Cartesian coordinate of the particle as an independent variable. A numerical example assuming the resistance force proportional to the square of the particle speed is presented. A review of existing results for related problems is provided, and it can be shown that these problems may be regarded as special cases of the brachistochrone problem formulated and solved in this paper under very general assumptions by means of optimal control theory.

1. Introduction

In 1696 Johann Bernoulli formulated the brachistochrone problem: find a smooth curve down which a particle slides from rest at a point *A* to a point *B* in a vertical plane influenced by its own gravity in the least time. After the brachistochrone problem had been independently solved by the Bernoulli brothers (Johann and Jacob), Newton, Leibnitz, Huygens and L'Hospital, various generalizations of the classical brachistochrone problem have been made. The brachistochronic motion of a particle was considered in various fields of active forces as well as under the action of various types of resistance forces (viscous friction forces, Coulomb friction force etc.). It is characteristic that the variational calculus method [1,2] was mainly used for various generalizations of the classical brachistochrone problem. However, from [3] to [4] and to [5], Pontryagin's minimum principle [6,7] and the singular optimal control theory [8] were included in solving this problem. Thus, in [3] the solution to the brachistochrone problem was realized within the theory of optimal control, while for the case of the brachistochrone in a resistant medium and the field of central forces the corresponding relations were developed only, representing the necessary optimality conditions. A numerical solution to the classical brachistochrone problem is given in [4]. An analytical solution to the problem of brachistochrone with Coulomb friction is presented in [5]. In other papers, generalizations of the problem were made using the variational calculus method: in the cases with Coulomb friction [9–13], where analytical solutions were obtained in [9–12] under various boundary conditions and various parameterizations of the brachistochrone curve, while in [13] the solution was obtained numerically; in the case with viscous friction, where as an independent variable was the particle speed in [14] and the slope angle of the brachistochrone curve in [15]; in the case with various fields of central forces in [16–19]. Among the mentioned types of generalizations of the brachistochrone problem, the paper [20] is interesting for its use of the Fermat principle from geometric optics. This principle was utilized

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