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Uncertainty assessment for inverse problems in high dimensional spaces using particle swarm optimization and model reduction techniques

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ABSTRACT

Global optimization methods including particle swarm optimization are usually used to solve optimization problems when the number of parameters is low (hundreds). Also, to be able to find a good solution typically involves multiple evaluations of the objective (or cost) function. Thus, both a large number of parameters and very costly forward evaluations hamper the use of global algorithms in inverse problems. In this paper, we address the first problem showing that the sampling can be performed in a reduced model space. The reduction of the dimension is accomplished in this case by the principal component analysis computed on a set of scenarios that are built based on prior information using stochastic simulation techniques. The use of a reduced base helps to regularize the inverse problem and to find a set of equivalent models that fit the data within a prescribed tolerance, allowing uncertainty analysis around the minimum misfit solution. We show the application of this idea to a history matching problem of a synthetic oil reservoir, using different members of the PSO family to perform sampling on the reduced model space.
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1. Inverse problems and particle swarm optimization

Inverse problems can be written in the discrete form as:

$$\mathbf{F}(\mathbf{m}) = \mathbf{d},$$

where $\mathbf{F}(\mathbf{m})$ is the forward operator that serves to account for the data prediction; $\mathbf{m} \in \mathbf{M} \subset \mathbf{R}^n$ are the model parameters, and $\mathbf{d} \in \mathbf{R}^s$ the discrete observed data. In some cases, the number of parameters n used to solve the forward problem is very high (several thousands) due to the fine discretization used in the model space in order to achieve accurate data predictions. This causes the inverse problem also to be highly ill-posed, that is, no unique solution exist and/or the inverse problem is very ill-conditioned. Ill-conditioning is an important issue when solving the inverse problem as an optimization problem, because noise in observed data is amplified back to the model parameters through the pseudoinverse forward operator, $\mathbf{F}^{\dagger}$.

Global optimization algorithms are a good alternative to the ill-conditioned character of inverse problems, because they approach the inverse problem as a sampling problem instead of looking for the pseudoinverse operator. Also, they only need as prior information the search space of possible solutions. Typically they use as cost (or objective) function, the data prediction misfit in a certain norm $p : \|\mathbf{F}(\mathbf{m}) - \mathbf{d}\|_p$. It is possible to show analytically that the topography of the cost function (without any regularization) is that of a non-convex problem; that is, there exists a family of equivalent models that fit equally the observed data within the same prescribed tolerance, *tol*. All these models are located along flat elongated

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