Mathematical and Computer Modelling 54 (2011) 2848–2857

Contents lists available at SciVerse ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Generalized higher order Stirling numbers

B.S. El-Desouky [a,*], Nenad P. Cakić [b]

[a] Department of Mathematics, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt

[b] Department of Mathematics, Faculty of Electrical Engineering, University of Belgrade, P.O. Box 35-54, 11120 Belgrade, Serbia

ARTICLE INFO

Article history:
Received 21 December 2010
Accepted 6 July 2011

Keywords:
Differential operators
Generalized multiparameter non-central Stirling numbers
Generalized Comtet numbers
Generalized Truesdel polynomials
Generalized Bell polynomials
Generalized Dobinski formula

ABSTRACT

In this paper new explicit expressions for both kinds of Comtet numbers and some interesting special cases are derived. Moreover, we define and study the generalized multiparameter non-central Stirling numbers and generalized Comtet numbers via differential operators. Furthermore, recurrence relations and new explicit formulas for those numbers are obtained. Finally some interesting special cases, new combinatorial identities and a connection between these numbers and some interesting polynomials are deduced.

1. Introduction

Let the falling factorial be defined by

$$(x)_n = x(x-1)\cdots(x-n+1), \quad (x)_0 = 1.$$

The generalized falling factorial $(t; \bar{\boldsymbol{\alpha}})_n$, associated with the parameter $\bar{\boldsymbol{\alpha}} = (\alpha_0, \alpha_1, \ldots, \alpha_{n-1})$ where $\alpha_j, j = 0, 1, \ldots, n-1$, is a sequence of real or complex numbers, is defined by

$$(t; \bar{\boldsymbol{\alpha}})_n = \prod_{j=0}^{n-1}(t - \alpha_j).$$

Note that if $\alpha_i = i\alpha, i = 0, 1, \ldots, n-1$, then $(t; \bar{\boldsymbol{\alpha}})_n$ reduces to $(t|\alpha)_n = t(t-\alpha)\cdots(t-(n-1)\alpha)$.

The multiparameter non-central Stirling numbers of the first and the second kind, respectively, were introduced by El-Desouky [1] with

$$(t)_n = \sum_{k=0}^{n} s(n, k; \bar{\boldsymbol{\alpha}})(t; \bar{\boldsymbol{\alpha}})_k \tag{1.1}$$

and

$$(t; \bar{\boldsymbol{\alpha}})_n = \sum S(n, k; \bar{\boldsymbol{\alpha}})(t)_k. \tag{1.2}$$

The numbers $s(n, k; \bar{\boldsymbol{\alpha}})$ and $S(n, k; \bar{\boldsymbol{\alpha}})$, respectively, satisfy the recurrence relations

$$s(n+1, k; \bar{\boldsymbol{\alpha}}) = s(n, k-1; \bar{\boldsymbol{\alpha}}) + (\alpha_k - n)s(n, k; \bar{\boldsymbol{\alpha}}) \tag{1.3}$$

* Corresponding author. Tel.: +20 50 2352440.
E-mail addresses: b_desouky@yahoo.com, s_belal@hotmail.com (B.S. El-Desouky), cakic@etf.rs (N.P. Cakić).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.07.005