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Fixed point theorems for multi-valued mappings obtained by altering distances

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ABSTRACT

Recently T. Suzuki showed that the Mizoguchi–Takahashi fixed point theorem is a real generalization of Nadler's fixed point theorem. Taking inspiration from the result of Mizoguchi and Takahashi and using the ideas of Feng and Liu, Klim and Wardowski obtained some fixed point theorems and showed that their results are different from the Reich point theorem and the Mizoguchi–Takahashi fixed point theorem. Very recently, Pathak and Shahzad introduced a class of functions and generalized some fixed point theorems of Klim and Wardowski by altering distances, i.e., via the mapping T (from a complete metric space (X, d) to the class of nonempty closed subsets of X). In this paper we introduce a new class of functions which is a subclass of the class introduced by Pathak and Shahzad and improve some results of Pathak and Shahzad by allowing T to have values in closed subsets of X.

1. Preliminaries

Let (X, d)be a metric space. For $x \in X$ and $A \subseteq X$, $d(x, A) = \inf\{d(x, y) : y \in A\}$. A subset A of X is called proximinal [1] if, for each $x \in X$, there is an element $a \in A$ such that $d(x, a) = d(x, A)$. We denote by $N(X)$ the class of all nonempty subsets of X, by $CL(X)$ the class of all nonempty closed subsets of X, by $PC(X)$ the class of all nonempty proximinal subsets of X, by $CB(X)$ the class of all nonempty closed and bounded subsets of X and by $K(X)$ the class of all nonempty compact subsets of X. Let H be the Hausdorff metric on $CL(X)$ generated by the metric d, that is,

$$H(A, B) = \begin{cases} \max\left\{\sup_{x\in A} d(x, B),\ \sup_{y\in B} d(y, A)\right\}, & \text{if the maximum exists} \\ \infty, & \text{otherwise} \end{cases}$$

for every $A, B \in CL(X)$. A point $p \in X$ is said to be a fixed point of $T : X \to CL(X)$ if $p \in Tp$. If, for $x_0 \in X$, there exists a sequence $\{x_n\}$ in X such that $x_n \in Tx_{n-1}$, then $O(T, x_0) = \{x_0, x_1, x_2, \ldots\}$ is said to be an orbit of $T : X \to CL(X)$. A mapping $f : X \to \mathbb{R}$ is said to be T-orbitally lower semi-continuous [2] if $\{x_n\}$ is a sequence in $O(T, x_0)$ and $x_n \to \xi$ implies $f(\xi) \le \lim_n \inf f(x_n)$. Suppose that $T : X \to N(X)$. Suppose that $A \in (0, +\infty]$ and let $\mathbb{R}$ denote the set of real numbers. $\Theta[0, A)$ [3] denotes the class of functions $\theta : [0, A) \to \mathbb{R}$ satisfying the following conditions: (i) θ is nondecreasing on $[0, A)$; (ii) $\theta(t) > 0$ for each $t \in (0, A)$; (iii) θ is subadditive in $(0, A)$, i.e., $\theta(t_1 + t_2) \le \theta(t_1) + \theta(t_2)$ for $t_1, t_2 \in (0, A)$. Notice that (i) implies that θ is strictly inverse isotone on $(0, A)$, i.e., $\theta(t_1) < \theta(t_2) \Rightarrow t_1 < t_2,\ t_1, t_2 \in (0, A)$. For $b \in (0, 1]$ and $x \in X$, we define $I_b^x = \{y \in Tx : bd(x, y) \le d(x, Tx)\}$, and $M(b, x; \theta) = \{y \in Tx : b\theta(d(x, y)) \le \theta(d(x, Tx))\}$.

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