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Inverse nodal problems for the p-Laplacian with eigenparameter dependent boundary conditions

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ABSTRACT

We study the issues of reconstruction of the inverse nodal problem for the one-dimensional p-Laplacian eigenvalue problem with eigenparameter boundary value conditions. A key step is the application of a modified Prüfer substitution to derive a detailed asymptotic expansion for the eigenvalues and nodal lengths. The parameter boundary data are also reconstructed.

1. Introduction

Recently, we have studied the inverse nodal problem for the p-Laplacian operator

$$-(y'^{(p-1)})' = (p-1)(\lambda - q(x))y^{(p-1)}, \tag{1.1}$$

coupled with separated or periodic boundary conditions. Here, $p > 1$, $q \in L^1(0,1)$ and $f^{(p-1)} \equiv |f|^{p-2}f$. We have showed that, using the information of the nodal data, the potential function q can be uniquely determined up to a constant and can be reconstructed [1,2]. In this paper, we will discuss the p-Laplacian operator (1.1) coupled with eigenparameter dependent boundary conditions

$$y(0) = 0, \qquad \alpha y'(1) + \lambda y(1) = 0, \tag{1.2}$$

where $\alpha \neq 0$ and λ is a real parameter.

For $p = 2$, Eq. (1.1) is the Sturm–Liouville equation

$$-y'' = (\lambda - q(x))y. \tag{1.3}$$

Consider Eq. (1.3) with eigenparameter dependent boundary conditions

$$(a_j\lambda + b_j)y(j) = (c_j\lambda + d_j)y'(j), \qquad (-1)^j(a_jd_j - b_jc_j) \leq 0, \quad j = 0, 1,$$

where $a_j^2 + b_j^2 + c_j^2 + d_j^2 \neq 0$. This problem differs from the usual regular Sturm–Liouville problem in the sense that the eigenvalue parameter λ is contained in the boundary condition. Problems of this type arise from applying the method of separation of variables to mathematical models for certain physical problems including heat and wave propagation. For

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