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Some further refinements and extensions of the Hermite–Hadamard and Jensen inequalities in several variables

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ABSTRACT

The main object of this paper is to give several refinements and extensions of the Hermite–Hadamard and Jensen inequalities in n variables. Relevant connections of the results presented here and the various inequalities derived in earlier investigations are also indicated.

1. Introduction

Let

$$\boldsymbol{x} = (x_1, \ldots, x_n) \quad \text{and} \quad \mathbb{N}_0 := \mathbb{N} \cup \{0\} \quad (\mathbb{N} := \{1, 2, 3, \ldots\}).$$

Also let $\mathbb{I}$ be a convex subset of an arbitrary real linear space $\mathbb{X}$. A function $f : \mathbb{I} \to \mathbb{R}$ is called convex if, for every two elements $a, b \in \mathbb{I}$, the following inequality holds true:

$$f\left(\frac{a+b}{2}\right) \leqq \frac{f(a)+f(b)}{2}. \tag{1.1}$$

We begin by recalling the following known results.

Theorem 1. (*see* [1,2]) *For every convex function f, the Jensen inequality:*

$$f\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \leqq \frac{1}{n}\sum_{i=1}^{n} f(x_i) \tag{1.2}$$

and the weighted Jensen inequality:

$$f\left(\frac{1}{P_n}\sum_{i=1}^{n} p_i x_i\right) \leqq \frac{1}{P_n}\sum_{i=1}^{n} p_i f(x_i) \tag{1.3}$$

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