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An electrostatic–elastic membrane system with an external pressure

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ABSTRACT

An elastic membrane subject to an external pressure and electrostatic force is considered. The resulting elliptic problem is analyzed for the strip and disk geometry. The structure of the solution set is explored with the aid of phase plane techniques, perturbation methods, and bifurcation analysis. In each case, the stability of the solution is determined using linear stability theory with the aid of the bifurcation diagram.

1. Introduction

In 1968, the British scientist Taylor published a paper studying the coalescence of fluid drops as they became close to each other [1]. He conjectured that there was a critical distance where the inert force between them would cause them to coalesce. During his work on this subject, a number of experiments were performed and a final setup was attained, which consisted of an elastic membrane (soap film) and a ground plate of which a potential difference was applied between. This voltage was meant to simulate the attractive force between the two drops. Applying this voltage causes the membrane to deflect toward the ground plate. You can envision the distance from the membrane to the ground plate as being half the distance from one liquid drop to the other. That is, the membrane touching the ground plate corresponds to the drops coalescing. Taylor observed that once the voltage reached a critical value there was no longer a stable configuration and the membrane contacted the ground plate. This instability in the system is often called the "pull-in" instability.

In Taylor's work he focused mainly on experimental results and did not take into account the membrane initially being deflected away from the ground plate. The disk geometry, which was the interest of [1], was further explored in [2] to determine maximal stable operating voltages and maximal stable deflections of the membrane. In [3], this problem was explored for the strip geometry. Later in [4], Pelesko and Goldsztein considered this problem in the context of MEMS devices. In their work, they consider both pressures that would deflect the membrane toward the ground plate and those that would move them away. An outline of the structure of the solution set to this problem is given for the strip geometry. It is observed that the structure of the solution set is greatly affected by the pressure. In this paper we confirm the structure of the solution set found in [4], as well as, analyze the stability of solutions for the strip geometry. Additionally, we consider the two-dimensional disk geometry with radial symmetry. We discuss the structure of the solution set and the stability of these solution. This will lead to a better understanding of the problem which will aid in the design process where these scenarios arise. The study of electrostatic–elastic membrane systems is important, not only in the field of electrohydrodynamics, but also in the study of electrostatic actuators and their importance to the design of MEMS devices where they are used.

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