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Finite volume difference scheme for a degenerate parabolic equation in the zero-coupon bond pricing

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ABSTRACT

In this paper we solve numerically a *degenerate* parabolic equation with *dynamical* boundary conditions of zero-coupon bond pricing. First, we discuss some properties of the differential equation. Then, starting from the divergent form of the equation we implement the finite volume method of Wang (2004) [6] to discretize the differential problem. We show that the system matrix of the discretization scheme is an M-matrix, so that the discretization is *monotone*. This provides the non-negativity of the price with respect to time if the initial distribution is non-negative. Numerical experiments demonstrate the efficiency of our difference scheme near the ends of the interval where the degeneration occurs.

1. Introduction

Since the Black–Scholes models rely on stochastic differential equations, option pricing rapidly became an attractive topic for specialists in the theory of probability and stochastic methods were developed first for practical applications, along with analytical closed formulas. But soon, with the rapidly growing complexity of the financial products, other numerical solutions became attractive [1–9].

There is a large and ever-going number of different interest rate derivative products now, for instance, bonds, bonds options, interest rate caps, swap options, etc. Bonds in general carry coupons, but there also exists a special kind of bond without coupons which is called zero-coupon bond (ZCB). A ZCB is purchased today at certain price, while at maturity the bond is redeemed for a fixed price. By a similar way to the derivation of the Black–Scholes equation, the problem of ZCB pricing can be reduced to a partial differential equation (see [10,11]).

The present paper deals with a degenerate parabolic equation of zero-coupon bond pricing [10,11]. Since our equation (see (1), (2), (3)) in the next section becomes *degenerate* at the boundary of the domain, classical finite difference methods may fail to give accurate approximations near the boundary. An effective method that resolves the singularity is proposed by Wang [6] for the Black–Scholes equation. The method is based on a finite volume formulation of the problem coupled with a fitted local approximation to the solution and an implicit time-stepping technique. The local approximation is determined by a set of two-point boundary value problems defined on the element edges. This fitting technique is based on the idea proposed by Allen and Southwell [12,13] for convection–diffusion equations and has been extended to one and multidimensional problems by several authors [14,12,13].

This paper is organized as follows. Our model problem is presented in Section 2, where we discuss our basic assumptions and some properties of the solution. The discretization method is developed in Section 3. Section 4 is devoted to the time discretization. We show that the system matrix is a M-matrix, so that the discretization is monotone. In this case the maximum principle is satisfied and thus the discrete solution is non-negative. Numerical experiments show higher accuracy

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