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One-sided Tauberian conditions for a general summability method

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ABSTRACT

Let (u_n) be a sequence of real numbers and L be an additive summability method with some property. We show that if slow decrease of (u_n) or one-sided boundedness of the classical control modulo of the oscillatory behavior of (u_n) is a Tauberian condition for a general summability method L, then one-sided boundedness by a sequence with certain conditions of the general control modulo of the oscillatory behavior of integer order m of (u_n) is also a Tauberian condition for L.

1. Introduction

Throughout this paper, let $u = (u_n)$ be a sequence of real numbers, and the symbols $u_n = o(1)$ and $u_n = O(1)$ mean that $u_n \to 0$ as $n \to \infty$ and that (u_n) is bounded for large enough n, respectively.

For a sequence (u_n),

$$u_n - \sigma_n^{(1)}(u) = V_n^{(0)}(\Delta u) \quad (n = 0, 1, 2, \ldots), \tag{1.1}$$

where $V_n^{(0)}(\Delta u) = \frac{1}{n+1}\sum_{k=0}^{n} k\Delta u_k$ and $\sigma_n^{(1)}(u) = \frac{1}{n+1}\sum_{k=0}^{n} u_k$. Note that $\Delta u_n = u_n - u_{n-1}$ and $u_{-1} = 0$.

The sequence $(\sigma_n^{(1)}(u))$ is called a sequence of $(C, 1)$ means of (u_n).

For each nonnegative integer m and for all nonnegative integers n, define $(\sigma_n^{(m)}(u))$ by

$$\sigma_n^{(m)}(u) = \begin{cases} \dfrac{1}{n+1}\displaystyle\sum_{k=0}^{n} \sigma_k^{(m-1)}(u), & m \geq 1 \\ u_n, & m = 0. \end{cases}$$

A sequence (u_n) is Abel summable to s if the limit $\lim_{x\to 1^-}(1-x)\sum_{n=0}^{\infty} u_n x^n = s$. More generally, a sequence (u_n) is L summable to s if $L - \lim_n u_n = s$. A summability method L is called additive if $L - \lim_n u_n = s$ and $L - \lim_n v_n = t$ imply that $L - \lim_n (u_n + v_n) = s + t$. A summability method L is called regular if the L-limit of every convergent sequence is equal to its limit. L is called $(C, 1)$ regular if $L - \lim_n u_n = s$ implies $L - \lim_n \sigma_n^{(1)}(u) = s$. For instance, Abel summability method is a regular summability method. It is clear that every regular summability method is $(C, 1)$ regular.

The classical control modulo of the oscillatory behavior of (u_n) is denoted by $\omega_n^{(0)}(u) = n\Delta u_n$, and the general control modulo of the oscillatory behavior of integer order $m \geq 1$ of a sequence (u_n) is defined inductively in [1,2] by

$$\omega_n^{(m)}(u) = \omega_n^{(m-1)}(u) - \sigma_n^{(1)}(\omega^{(m-1)}(u)).$$

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