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# A note on the parabolic equation with an arbitrary parameter at the derivative

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ABSTRACT

We consider the parabolic differential equation

$$\varepsilon u'(t) + Au(t) = f(t), -\infty < t < \infty, \tag{0.1}$$

in a Banach space $E$ with a strongly positive operator $A$ and with an arbitrary positive parameter $\varepsilon$. We establish the well-posedness in difference analogue of Hölder space of the high order uniform difference scheme for (0.1). Moreover, in applications, the convergence estimates for the solutions of uniform difference schemes of the multi-dimensional parabolic differential equations with an arbitrary positive parameter $\varepsilon$ are obtained.



## 1. Introduction

In the study of boundary value problems for parabolic and elliptic differential equations, the role of well-posedness (maximal regularity, coercivity inequalities) is well known (see e.g. [1–10] and the references therein).

In [5], the Cauchy problem

$$\varepsilon u'(t) + Au(t) = f(t), \quad 0 < t < T, \qquad u(0) = u_0 \tag{1.1}$$

is investigated. It is well known that no classical finite difference approximations of (1.1) yield uniform convergence with respect to $\varepsilon$. This is a rather disappointing result and hence it naturally raises the question about the existence of any difference schemes which are uniform in $\varepsilon$ (see [8, Chapter 7]). Various problems for the parabolic equation with small positive parameter $\varepsilon \in (0, 1)$ have been investigated in [5,6,8,11–19]. In [5], the high order of accuracy single-step uniform difference schemes of the approximate solutions of (1.1) are constructed.

In the present paper, we consider the parabolic differential equation

$$\varepsilon u'(t) + Au(t) = f(t), \quad -\infty < t < \infty \tag{1.2}$$

in an arbitrary Banach space $E$ with a strongly positive operator $A$ and with an arbitrary positive parameter $\varepsilon \in (0, \infty)$. Here, $u(t)$ and $f(t)$ are unknown and given abstract functions defined on $\mathbb{R}$ with values in $E$, and $A$ is a densely defined linear unbounded closed operator acting in $E$.

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