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Blow-up solutions of nonlinear Volterra integro-differential equations

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ABSTRACT

The paper studies the finite-time blow-up theory for a class of nonlinear Volterra integro-differential equations. The conditions for the occurrence of finite-time blow-up for nonlinear Volterra integro-differential equations are provided. Moreover, the finite-time blow-up theory for nonlinear partial Volterra integro-differential equations with general kernels is also established using the blow-up results for the nonlinear Volterra integro-differential equations.

1. Introduction

The theory of finite-time blow-up solutions has been well studied for nonlinear Volterra integral equations (see [1–5]). However, to the best of our knowledge, there are no analogous results for nonlinear Volterra integro-differential equations (VIDEs). In this paper, we study the finite-time blow-up theory for nonlinear VIDEs and nonlinear partial Volterra integro-differential equations (PVIDEs).

In the first part of this paper, we study VIDEs of the form:

$$y'(t) = -ay(t) + \int_0^t k(t-s)g(y(s))\,\mathrm{d}s, \quad t > 0, \tag{1}$$

with $y(0) = y_0 \geq 0$, where we assume that a is a nonnegative constant and

(a) $k(t)$ is an integrable positive function such that $\lim_{t\to\infty} K(t) = \infty$, where $K(t) = \int_0^t k(s)\,\mathrm{d}s$,
(b) $g(t)$ is nonnegative, nondecreasing and continuous for $t > 0$, $g \equiv 0$ for $t \leq 0$, and

$$\lim_{y\to\infty} \frac{g(y)}{y} = \infty.$$

The finite-time blow-up theories for Eq. (2) and more general types of nonlinear Volterra integral equations are established in [1–6]. We know that Brunner and Yang [6] have recently developed a new technique to investigate the blow-up theories for VIEs and partially for VIDEs including Eq. (1) with $a < 0$. However, the case (1) with $a > 0$, which is more difficult, is not studied in their paper. In this paper we investigate problem (1) with $a > 0$.

For $a \equiv 0$ and $y_0 \equiv 0$, Eq. (1) can be easily converted into a VIE

$$y(t) = \int_0^t h(t-s)g(y(s))\,\mathrm{d}s, \tag{2}$$

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