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 **ScienceDirect**  
Journal of Hydrodynamics

2011,23(1):1-11

DOI: 10.1016/S1001-6058(10)60081-9



[www.sciencedirect.com/science/journal/10016058](http://www.sciencedirect.com/science/journal/10016058)

# Chebyshev Finite Spectral Method for 2-D Extended Boussinesq Equations\*

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(Received December 16, 2010, Revised January 4, 2011)

**Abstract:** In this article, an accurate Chebyshev finite spectral method for the 2-D extended Boussinesq equations is proposed. The method combines the advantages of both the finite difference and spectral methods. The Adams-Bashforth predictor and the fourth-order Adams-Moulton corrector are adopted for the numerical solution of the governing differential equations. An efficient wave absorption strategy is also proposed to effectively absorb waves at outgoing wave boundaries and reflected waves from the interior of the computational domain due to barriers and bottom slopes at the incident wave boundary to avoid contamination of the specified incident wave conditions. The proposed method is verified by a case where experimental data are available for comparison for both regular and irregular waves. The case is wave diffraction over a shoal reported by Vincent and Briggs. Numerical results agree very well with the corresponding experimental data.

**Key words:** Chebyshev polynomial, finite spectral method, irregular waves

## Introduction

The propagation of surface waves is of fundamental and practical importance in oceanography and marine engineering. As waves propagate toward shore, the combination of shoaling, refraction, and diffraction effects will modify their waveforms. The study of the transformation process can be based on either the Navier-Stokes (N-S) equations<sup>[1]</sup>, mild slope equation or Boussinesq equations.

The Boussinesq-type equations, which are weakly non-linear and dispersive, have been found to give a relatively accurate description of the phenomenon of wave transformation in coastal

regions, including the energy transfers among the different components of the shoaling wave, provided the underlying assumptions in their derivation are not violated. These 2-D equations are obtained by the integration of the 3-D N-S equations over the water depth, making their solutions tractable because of a significant reduction in computation time. The first set of such equations was derived by Peregrine<sup>[2]</sup>, which are referred to as the standard Boussinesq equations. However, the standard Boussinesq equations are only applicable in shallow water because of the inaccuracy of the linear dispersion relation with increasing water depth.

To extend their range of applicability, many investigators have suggested various improvements to the standard Boussinesq equations or derived alternative forms. Nwogu<sup>[3]</sup> derived an alternative set of Boussinesq equations in terms of the velocities at an arbitrary level. Beji and Nadaoka<sup>[4]</sup> gave a set of Boussinesq equations with exact energy conservation by adding and subtracting a dispersive term in the momentum equations. Kim et al.<sup>[5]</sup> developed a

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\* Project supported by the Research Grants Council of the Hong Kong Special Administrative Region, China (Grant No. PolyU5220/07E), the National Marine Public Welfare Research Projects of China (Grant No. 201005002).

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