



## A HIGHER-ORDER NON-HYDROSTATIC MODEL FOR SIMULATING WAVE PROPAGATION OVER IRREGULAR BOTTOMS\*

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**Abstract:** A higher-order non-hydrostatic model is developed to simulate the wave propagation over irregular bottoms based on a vertical boundary-fitted coordinate system. In the model, an explicit projection method is adopted to solve the unsteady Euler equations. Advection terms are integrated explicitly with the MacCormack's scheme, with a second-order accuracy in both space and time. Two classical examples of surface wave propagation are used to demonstrate the capability of the model. It is found that the model with only two vertical layers could accurately simulate the motion of waves, including wave shoaling, nonlinearity, dispersion, refraction, and diffraction phenomena.

**Key words:** non-hydrostatic, wave propagation, projection method, boundary-fitted coordinate

### Introduction

An accurate simulation of wave propagation over irregular bottoms is important to ocean and coastal engineering. The Boussinesq-type models<sup>[1,2]</sup> were widely used for simulating water wave motions. They are the so-called depth-integrated models and can be derived by integrating Euler equations over the water depth. Although many depth-averaged models can be successfully applied to simulate short wave motions, they can not predict the vertical flow structure. In addition, the highly nonlinear and dispersive Boussinesq models are discretized in a complicated way with a large computational cost. A 3-D non-hydrostatic model as a unified model is capable of simulating water wave motions successfully.

The so-called non-hydrostatic model based upon the Euler equations (or Navier-Stokes equations) with the free surface equation to capture the water surface is the most detailed approach for simulating water motions. In recent years, many non-hydrostatic models were developed<sup>[3-8]</sup>. Stelling and Zijlema<sup>[3,4]</sup> proposed a Keller-box scheme for the approximation of the vertical gradient of the non-hydrostatic pressure at the cell faces of the vertical grids. Satisfactory results

were obtained with only two vertical layers, and their model could be used to predict short wave motions in an accurate and efficient manner. Yuan and Wu<sup>[5]</sup> proposed an integral method to remove the assumption of the top-layer hydrostatic distribution. Their results show that with a very small number of vertical layers, a range of free-surface flow problems can be accurately simulated.

This article proposes a higher-order non-hydrostatic model for the simulation of wave propagation. Two test cases, including (1) a wave propagation over a submerged bar, and (2) a 3-D wave refraction and diffraction over an elliptic shoal, are employed to demonstrate the capability of the model.

### 1. Governing equations

The free surface water flow is governed by the incompressible Euler equations, based on the conservation of mass and momentum, which can be written in vector forms as follows

$$\nabla \cdot \mathbf{U} = 0 \quad (1)$$

and

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = -\mathbf{P} \quad (2)$$

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