A new nonreflecting open boundary condition for circular cavities in unbounded domain

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Abstract

In this paper, a new non-reflecting open boundary condition is introduced in order to solve circular cavity problems. In this regard, a new semi-analytical method is used to diagonalize the coefficient matrices of governing differential equation. This method uses four tools in order to diagonalize the coefficient matrices in which they are: higher-order sub-parametric elements, higher-order Chebyshev mapping functions, weighted residual method and Clenshaw-Curtis quadrature. This method is developed to diagonalize the dynamic stiffness matrix. To this aim, the substructure method is used. Two first order ordinary differential equations (i.e. interaction force-displacement relationship and governing differential equation in dynamic stiffness) are solved to satisfy the radiation condition at infinity and boundary condition of soil-structure interface. The interaction force-displacement relationship is considered as a nonreflecting open boundary condition for the bounded medium substructure. Two cavities embedded in a full-plane are considered as benchmark examples and the results are compared with analytical solutions.

Keywords: semi-analytical method, open nonreflecting boundary condition, Chebyshev polynomial, sub-parametric elements.

١. INTRODUCTION

The dynamic analysis of medium-structure interaction based on substructure method requires investigating the dynamic response of unbounded media. In this regard, the dynamic stiffness matrix should be determined in the frequency domain. The dynamic property of unbounded domain could be illustrated by force-displacement relationship formulated at the medium-structure interface. In the substructure method, this relationship could be regarded as a Boundary Condition (BC) for bounded substructure.

Excellent literature reviews about open BCs are discussed in [١-٤]. The higher order open boundaries increase the accuracy as their order of approximation increases [٥]. Also, the formulations of these BCs are temporally local and are singly asymptotic at the high frequency limit. The doubly asymptotic boundaries are introduced to model an unbounded domain with the presence of nonradiative wave fields [٦-٩]. Various local BCs have been proposed [١٠-١١]. These BCs are simple, approximate and should be generally applied to a boundary sufficiently far from the region of interest.

In the frequency domain analysis, the dynamic stiffness matrix of the unbounded medium is required to be determined. The soil stiffness matrix may be derived by implementing FEM [١٢], BEM [١٣], SEM [١٤] and SBFEM [١٥].

The goal of this paper is to introduce a new semi-analytical method for diagonalization of dynamic stiffness matrix and proposing a new non-reflecting BC for circular cavity problems. The present method has been recently developed for solving potential [١٦], elastostatic [١٧, ١٨] and elastodynamic problems in the time [١٩] and the frequency [٢٠] domains. In this method, only the boundaries of the problem's domain are discretized with higher-order sub-parametric elements. Using the weighted residual method and implementing Clenshaw-Curtis quadrature leads to a diagonal system of Bessel's differential equation, in the frequency domain. In other words, the governing differential equation for each DOF is independent from other DOFs. In this method, the far field radiation BC at infinity is satisfied, exactly.

٢. SUMMARY OF NEW SEMI-ANALYTICAL METHOD

In order to explain the principals of the new semi-analytical method, an unbounded domain as shown in figure ١ in considered.