Modified Hardy-Cross methods with fifth-order convergence

Majid Niazkar[۱] and Seied Hosein Afzali[۲]
۱- PhD. Student of Civil Engineering,
۲- Assistant professor of Civil Engineering,
Dept. of Civil Eng., School of Engineering, Shiraz University

afzali@shirazu.ac.ir

Abstract

In this study, a new modified fifth-order Hardy-Cross methods is presented for water distribution systems. In the steady-state condition, the governing equations of water networks are the continuity equations for each node and the energy equations for each loop. The former equations are linear in respect to flow rate while the latter are nonlinear algebraic ones. Therefore, the true meaning of solving a pipe network is to solve a system of nonlinear algebraic equations in terms of pipe flow rates. On the contrary, the Hardy-Cross method, as one of the hydraulic-solver approaches, solves pipe networks without building the aforementioned system of equations, which is supposed to be more efficient than the matrix-based ones. Although the original Hardy-Cross method does not provide a matrix-based scheme, it performs much less efficient in comparison with the other matrix-based approaches. The Hardy-Cross method utilizes a specific kind of initial guess which satisfies the continuity equations in advance of dealing with the energy equations, which are individually solved for each loop. Hence, one of the major challenges of this approach is its low rate of convergence. The proposed modified Hardy-Cross methods grasp higher order of convergence than the original approach. Finally, the comparison of the proposed fifth-order Hardy-Cross methods with the original one clearly demonstrates that the recommended modification improves the rate of convergence of the original Hardy-Cross method.

Keywords: Water distribution system, pipe network, hydraulic solver, Hardy-Cross method, fifth-order convergence

۱. INTRODUCTION

Water distribution systems are one of major facilities in human civilizations. These systems take the burden of transferring and delivering sufficient amount of qualified water in time and space. The problems attributed to these systems become more and more vital with the increase of population and water demand. These problems have a wide range of varieties including analyzing, designing, optimizing, management, calibration, aging and rehabilitation. Regardless of the type of water network problems, analysis inevitably comprises an essential part of the problem. Subsequently, the literature of water distribution networks has mostly been devoted to the analyzing problems since contributions or improvement of this part will definitely affect other kinds of problems.

The analysis of water distribution network can be conducted in three different conditions: (۱) Steady-state condition, (۲) Extended-period simulation, (۳) Transient condition. The difference of these conditions is exclusively in the rate of change of the state variables of water networks, i.e., pipe flow rate (Q) and nodal hydraulic head (h), with time. In the first condition, the temporal variation of state variables is not considered while abrupt changes of state variables are modeled in the third condition. Despite of these two extreme conditions, the second condition is the moderate one. In this condition, the time of network analysis is divided into several intervals in which steady state condition governs in each interval. Hence, the modification of analyzing water network in the steady state condition can be extended to the extended-period simulation as well. In this research, the water network analysis is focused on the steady state condition.

In the steady state condition, the governing equations are: (۱) Water continuity equation and (۲) Energy equation. The unknown state variables of a pipe network can be determined by satisfying these two equations. The iteration-based methods of water distribution networks can be classified as two main approaches: (۱) Matrix based and (۲) Non-matrix based methods. The former ones form a matrix equation comprising the governing equations and attempt to solve these equations simultaneously whereas the latter deals with solving equations separately in two steps: First linear continuity equations are satisfied and second nonlinear energy equations are solved iteratively. The former method includes Linear Theory, Newton-