A new iterative method for solving free boundary problems

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Abstract

In this paper, an efficient iterative method is proposed to approximate the solution of free boundary problems (FBP). This method is based on hybrid of the radial basis function (RBF) collocation and finite difference (FD) methods. Finally, a numerical example is given to illustrate the good performance of the new method.

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1 Introduction

A free-boundary problem is a partial differential equation that in which some part of the boundary is not known, but is to be determined. The segment Γ of the boundary of domain which is not known is called the free boundary. Then, both the free boundary and the solution of the differential equation should be determined.

Let Ω be a bounded open subset of $\mathbb{R}^n$, $n \geq 1$ with smooth boundary $\partial\Omega$. Assume further that $g \in W^{1,2}(\Omega)$ and takes both positive and negative values over $\partial\Omega$, and $\lambda^{\pm} : \Omega \to \mathbb{R}$ are positive Lipschitz-continuous functions. The study of the following FBP is suggested by Weiss in [3]. Find a weak solution $u \in W^{1,2}(\Omega)$ of $\Delta u = \lambda^+\chi_{\{u>0\}} - \lambda^-\chi_{\{u<0\}}$, in Ω such that $u - g \in W_0^{1,2}(\Omega)$ for a given $g \in W^{1,2}(\Omega)$, where χ_A denotes the characteristic function of the set A. This problem can be modeled as follows

$$\begin{cases} \Delta u = \lambda^+\chi_{\{u>0\}} - \lambda^-\chi_{\{u<0\}}, & x \in \Omega; \\ u = g, & x \in \partial\Omega. \end{cases} \tag{1}$$

If, in addition, we assume that $\lambda^- = 0$ and g be non-negative on the boundary then we have the one-phase obstacle problem.
The free boundary problems (1) have been studied from different viewpoints, see [1, 2].

In this study, we propose an efficient iterative method to solve two-phase problem, one-phase obstacle problem and FBP of the form

$$\begin{cases} \Delta u = -\begin{cases} \lambda^+ u, & if\ u > 0; \\ 0 & if\ u \leq 0. \end{cases}, & x \in \Omega; \\ u = g, & x \in \partial\Omega. \end{cases}$$

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