Spectral solutions of time fractional telegraph equations

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Abstract

In this paper, A spectral scheme is proposed to approximate the solution of time fractional telegraph equations. Eigenfunctions of second order self-adjoint differential operator are used for discretization of spatial variable and Shifted Legendre polynomials are applied to discretization of time variable. Numerical results are presented for some problems to demonstrate the usefulness and accuracy of this approach. The method is easy to apply and produces very accurate numerical results.

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1 Introduction

Consider time fractional telegraph equation as

$$D_c^{\beta}U(x,t) + k_1 D_c^{\alpha}U(x,t) + k_2 U(x,t) = \frac{\partial^2 U}{\partial x^2}(x,t) + f(x,t), \tag{1}$$
$$(x,t) \in [0,1] \times [0,1], \quad 0 < \alpha \leq 1 < \beta \leq 2,$$

subject to the homogeneous boundary condition

$$U(x,0) = U(x,1) = 0, \tag{2}$$

and the initial condition

$$U(x,0) = f_0(x), \quad U_t(x,0) = f_1(x), \tag{3}$$

which k_1 and k_2 are constant and D_c^{α} is the Caputo-type fractional derivative of order α. These equations , when $\beta = 2$ and $\alpha = 1$, are commonly used in the study of wave propagation of electric signals in a cable transmission line and also in wave phenomena. And also they have been used in modeling the reactiondiffusion processes in various branches of engineering sciences and biological sciences by many researchers (see [1] and references therein).

The advantage of fractional derivatives [2, 3] become apparent in modeling mechanical and electrical properties of real materials. Many phenomena in fluid mechanics, viscoelasticity, chemistry, physics, finance and other sciences can be described very successfully by

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