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Numerical solution of an inverse source problem of the time-fractional diffusion equation using a LDG method

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Abstract

This paper is devoted to determine a time-dependent source term in a time-fractional diffusion equation using a fully discrete local discontinous Galerkin (LDG) method. This method is based on a finite difference scheme in time and a local discontinous Galerkin method in space, is numerically stable and has the convergence of order $O((\Delta x)^{k+1} + (\Delta t)^2 + (\Delta t)^{\frac{\alpha}{2}}(\Delta x)^{k+\frac{1}{2}} + (\Delta t)^{\alpha})$.

Keywords: LDG method, time-fractional diffusion equation, inverse source problem.
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1 Introduction

In this paper, we consider the following initial-boundary value problem for the time-fractional diffusion equation

$$\begin{cases} D_t^{\alpha} u = u_{xx} + f(x)p(t), & 0 < x < 1, \quad 0 < t < T, \\ u(0,t) = k_0(t), & \qquad\qquad 0 \leqslant t \leqslant T, \\ u(1,t) = k_1(t), & \qquad\qquad 0 \leqslant t \leqslant T, \\ u(x,0) = \phi(x), & 0 \leqslant x \leqslant 1. \end{cases} \tag{1}$$

Problem (1) is a forward problem when all of the functions f, ϕ, k_0, k_1 and p are given appropriately. The inverse source problem which is considered here is to determine the source term p based on problem (1) and the following additional condition

$$u(x^*, t) = g(t), \qquad 0 \leq t \leq T,$$

where $x^* \in (0,1)$ is an interior measurement location. D_t^{α} is the Caputo fractional derivatives of order α, i.e.

$$D_t^{\alpha} u = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial u(\cdot, s)}{\partial s} \frac{ds}{(t-s)^{\alpha}}, \qquad 0 < \alpha < 1, \tag{2}$$

where $\Gamma(.)$ is the Gamma function. The inverse source problem mentioned above has been solved numerically by Wei et al. [1] using a regularized method. We aim to apply the discontinuous Galerkin method to the above mentioned inverse source problem. Of

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