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Higher nummerical ranges of basic $A-$factor block circulant matrix

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Abstract

In this paper, using the notion of $k-$numerical range, the relation between $k-$numerical range of matrix polynomials and the $k-$numerical range of its linearization are investigated. Moreover, the $k-$numerical ranges of basic circulant $A-$factor matrix are studied.

Keywords: $k-$numerical range, matrix polynomial, companion linearization, basic $A-$factor block circulant matrix

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1 Introduction

Let k and n are positive integers, $\mathbb{M}_n$ be the algebra of all $n \times n$ complex matrices, The set of all $n \times k$ isometry matrices is denoted by $\mathcal{X}_{n\times k}$, i.e., $\mathcal{X}_{n\times k} = \{X \in \mathbb{M}_{n\times k} : X^*X = I_k\}$ and the group of $n \times n$ unitary matrices is denoted by $\mathcal{U}_n$. The $k-$numerical range of $A \in \mathbb{M}_n$ is defined and denoted by $W_k(A) = \{\frac{1}{k}tr\,(X^*AX) \;:\; X \in \mathcal{X}_{n\times k}\}$, where $tr(.)$ denotes the trace. The sets $W_k(A)$, where $k \in \{1, 2, \dots, n\}$, are generally called *higher numerical ranges* of A. Let $A \in \mathbb{M}_n$ have eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$, counting multiplicities. The set of all $k-$averages of eigenvalues of A is denoted by $\sigma^{(k)}(A)$, namely,

$$\sigma^{(k)}(A) = \{\frac{1}{k}\,(\lambda_{i_1} + \lambda_{i_2} + \cdots + \lambda_{i_k}) \;:\; 1 \le i_1 < i_2 < \cdots < i_k \le n\}.$$

Proposition 1.1. *Let $A \in \mathbb{M}_n$. Then the following assertions are true:*
(i) $W_k(A)$ is a compact and convex set in $\mathbb{C}$;
(ii) conv $\left(\sigma^{(k)}(A)\right) \subseteq W_k(A)$, The equality holds if A is normal;
(iii) $\{\frac{1}{n}tr(A)\} = W_n(A) \subseteq W_{n-1}(A) \subseteq \cdots \subseteq W_2(A) \subseteq W_1(A) = W(A)$;
(iv) If $V \in \mathcal{X}_{n\times s}$, where $k \le s \le n$, then $W_k\,(V^*AV) \subseteq W_k(A)$. The equality holds if $s = n$, i.e., $W_k\,(U^*AU) = W_k(A)$, where $U \in \mathcal{U}_n$;
(v) For any $\alpha, \beta \in \mathbb{C}$, $W_k(\alpha A+\beta I_n) = \alpha W_k(A)+\beta$, and for the case $k < n$, $W_k(A) = \{\alpha\}$ if and only if $A = \alpha I_n$;

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