Function-valued Gram-Schmidt process in $L_2(0, \infty)$

Mohammad Ali Hasankhani Fard*
Vali-e-Asr University of Rafsanjan

Abstract

In this paper, we will look at the Gram-Schmidt process corresponding to a function valued inner product in $L_2(0, \infty)$.

Keywords: function-valued inner product, function-valued norm, function-valued orthogonal, Gram-Schmidt process.
Mathematics Subject Classification [2010]: 42C15

1 Introduction

A function-valued inner product on $L_2(0, \infty)$ by using of the dilation operator and its application in dilation-invariant systems has been introduced in [3]. Fix $a > 1$. For each pair $f, g \in L_2(0, \infty)$, the function $\langle f, g \rangle_a$ on $(0, \infty)$ is defined by

$$\langle f, g \rangle_a (x) := \sum_{j \in \mathbb{Z}} a^j f(a^j x) \overline{g(a^j x)}$$

and is called function-valued inner product on $L_2(0, \infty)$ with respect to a. It is easy to show that $\langle f, g \rangle = \int_1^a \langle f, g \rangle_a (x) dx$, where $\langle ., . \rangle$ is the original inner product in $L_2(0, \infty)$. Also, the function-valued norm on $L_2(0, \infty)$ with respect to a is defined by

$$\|f\|_a(x) := \sqrt{\langle f, f \rangle_a (x)}, \quad \forall f \in L_2(0, \infty) \quad and \quad \forall x \in (0, \infty).$$

The function ϕ on $(0, \infty)$ is called dilation periodic function with period a if $\phi(ax) = \phi(x)$ for all $x \in (0, \infty)$. The set of bounded dilation periodic functions on $(0, \infty)$ is denoted by B_a. For any function ϕ on $[1, a]$, the function $\widetilde{\phi}$ defined by $\widetilde{\phi}(a^j x) = \phi(x)$, for all $j \in \mathbb{Z}$ and $x \in [1, a]$ is dilation periodic. Throughout this paper, let $\widetilde{\phi}$ be the dilation periodic function defined as above for any complex function ϕ on $[1, a]$. A function f defined on $(0, \infty)$ is called function-valued bounded respect to a, or simply function-valued bounded, if there is a $B > 0$ such that $\|f\|_a(x) \leq B$ for almost all $x \in [1, a]$. The set of function-valued bounded functions denote by $L_a^\infty(0, \infty)$.
The properties of the function-valued inner product are given in the next theorem.

*Speaker