Capability of groups satisfying a certain bound for the index of the center

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Abstract

It is shown that for a capable group G, the index of the center is bounded above by the function $|G'|^{2log_2|G'|}$. In this talk, we intend to determine the sufficient conditions for capability of a group G which satisfies this inequality.

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1 Introduction

In 1938, Bare[1] initiated a systematic investigation of the question which conditions a group G must fulfill in order to be the group of inner automorphisms of some group E ($G \cong E/Z(E)$). Following M. Hall and Senior [5] such a group G is called *capable*. Baer classified capable groups that are direct sums of cyclic groups. His characterisation of finitely generated abelian groups that are capable is given in the following theorem.

Theorem 1.1. *[1]. Let G be a finitely generated abelian group written as $G = \mathbb{Z}_{n_1} \oplus \mathbb{Z}_{n_2} \oplus \cdots \oplus \mathbb{Z}_{n_k}$, such that each integer $n_i + 1$ is divisible by n_i, where $\mathbb{Z}_0 = \mathbb{Z}$, the infinite cyclic group. Then G is capable if and only if $k \geqslant 2$ and $n_{k-1} = n_k$.*

In 1940, P. Hall [4] introduced the concept of isoclinism of groups, which is one of the most significant methods for classification of groups. He showed that capable groups play an important role in characterizing p-groups. Also, capability has interesting connections to other branch of group theory. So some authors studied different aspects of capable groups. One of the interesting aspects is finding a relation between the index of $Z(G)$ and the order of $G^{'}$ in a capable group G.

Understanding the relationship between $G/Z(G)$ and G' goes back at least to 1904 when I. Schur[11] proved that the finiteness of $G/Z(G)$ implies the finiteness of G'. Infinite extra-special p-groups show that the converse of Schur's theorem does not hold in general. Isaacs [6] proved that if G is a finite capable group, then $|G/Z(G)|$ is bounded above by a function of $|G^{'}|$. Podoski and Szegedy [8] extended Isaac's result and gave the following explicit bound for the index of the center in a capable group.

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