Multiwavelets Galerkin method for solving linear control systems

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Abstract

In this paper a numerical technique is proposed for solving linear control systems. Multiwavelets Galerkin method is applied for solving the extreme conditions obtained from the Pontryagin's maximum principle.

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1 Introduction

Optimal control theory has many successful practical applications in areas ranging from economics to various engineering disciplines. The optimal control problem has been studied by many researchers [1]. In this paper, we consider linear optimal problem (OCP)

$$\begin{aligned} &\dot{x} = Ax(t) + Bu(t), x(t_0) = x_0, \\ &J = \tfrac{1}{2}x(t_f)^T Sx(t_f) + \tfrac{1}{2}\int_{t_0}^{t_f}(x^T Px + 2x^T Qu + u^T Ru)dt, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times n}$. The control $u(t)$ is an admissible control if it is piecewise continuous in t for $t \in [t_0, t_f]$. The input $u(t)$ is derived by minimizing the quadratic performance index J, where $S \in \mathbb{R}^{n\times n}$, $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times m}$ are positive semi-definite matrices and $R \in \mathbb{R}^{m\times m}$ is positive definite matrix.

2 Optimality conditions for linear optimal control system

In this section, we try to get the optimal control law $u^*(t) = -k(t)x(t)$ for system (1) by using PMP [2]. For this purpose, one can consider Hamiltonian as

$$H(x, u, \lambda, t) = \frac{1}{2}(x^T Px + 2x^T Qu + u^T Ru) + \lambda^T(Ax + Bu), \tag{2}$$

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