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Some  $\mu$ -fixed point theorems for  $\mu$ -continuous Maps on  $\sigma$ -algebras

## Some $\mu$ -Fixed Point Theorems for $\mu$ -continuous Maps on $\sigma$ -algebras

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## Abstract

Let  $\mu$  be a positive finite measure on a  $\sigma$ -algebra  $\mathcal{A}$ . In this paper, we introduce the concept of  $\mu$ -fixed point for mappings  $f : \mathcal{A} \to \mathcal{A}$  and obtain conditions for the existence of common  $\mu$ -fixed points of such mappings. We show that for any  $\mu$ -complete measure space if there exist  $m \in \mathbb{N}$  and  $0 \le k < 1$  such that for every  $A \in \mathcal{A}, \mu(f^m(A)) \le k \mu(A)$ , then all  $\{f^n\}_{n \in \mathbb{N}}$  have a unique common  $\mu$ -fixed point.

**Keywords:** fixed point theorem, contraction mapping, measure **Mathematics Subject Classification [2010]:** 47H10, 47H09

## 1 Introduction

Schauder fixed point theorem states that any compact map from a nonempty, closed, convex, bounded subset E of a Banach space into itself has a fixed point in E [3]. Darbo [4] extended Schauder's fixed point theorem to the setting of noncompact operators by using the concept of  $\alpha - k$ -set contraction, where  $0 \le k < 1$  and  $\alpha$  denotes the Kuratowski measure of noncompactness [6]. In fact, he proved the following theorem.

**Theorem 1.1.** Let X be a nonempty, closed, bounded and convex subset of a Banach space and  $f: X \to X$  be a bounded continuous map with

$$\alpha(f(B)) \le k \; \alpha(B)$$

for all bounded subsetes B of X, where  $0 \le k < 1$ . Then f has a fixed point.

Sadovski [7] proved that above theorem is true for a bounded continuous map f such that

$$\alpha(f(B)) \le \alpha(B)$$

for all bounded subsets B of X with  $\alpha(B) > 0$ . Banas [2] proved a fixed point result using the concept of  $\beta - k$ -set contraction, where  $0 \le k < 1$  and  $\beta$  denotes the De Blasi measure of weak noncompactness [5]. Amini-Harandi, Fakhar and Zafarani [1] have introduced a type of generalized set contraction in topological spaces with respect to a measure of noncompactness and proved a fixed point theorem which are either generalized set contraction or condensing ones.

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