46th Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

Block matrix operators and p-paranormality

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Abstract

In this paper we introduce a new model of a block matrix operator $M(\gamma, \eta)$ induced by two sequences γ and η. Then by its corresponding composition operator C_T on $\ell^2_+ = L^2(\mathbb{N}_0)$ we characterize p-paranormality the block matrix operator $M(\gamma, \eta)$.

Keywords: p-paranormal operator, composition operator, conditional expectation.
Mathematics Subject Classification [2010]: 47B20, 47B38

1 Introduction

Let $\mathcal{H}$ be the infinite dimensional complex Hilbert space and $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$ and let $T = U|T|$ be the canonical polar decomposition for $T \in \mathcal{L}(\mathcal{H})$. An operator $T \in \mathcal{L}(\mathcal{H})$ is said to be p-paranormal if $\||T|^pU|T|^px\| \geq \||T|^px\|^2$, for all unit vectors $x \in \mathcal{H}$. By using the property of read quadratic forms T is p-paranormal operator if and only if for all integers $k \geq 0$, $|T|^pU^*|T|^{2p}U|T| - 2k|T|^{2p} + k^2 \geq 0$.

Let (X, Σ, μ) be a complete σ-finite measure space and let $T : X \to X$ be a transformation such that $T^{-1}(\Sigma) \subseteq \Sigma$ and $\mu \circ T^{-1} \ll \mu$. It is assumed that the Radon-Nikodym derivative $h = d\mu \circ T^{-1}/d\mu$ is in $L^\infty(X)$. The composition operator C_T on $L^2(X)$ is defined by $C_Tf = f \circ T$. The condition $h \in L^\infty(X)$ assures that C_T is bounded. All comparisons between two functions or two sets are to be interpreted as holding up to a μ-null set. In [3] Jabbarzadeh and Azimi characterize p-paranormality of C_T on $L^2(X)$. A key tool in [3] was the use of the conditional expectation operators for studying p-paranormality of C_T, and this will be the main tool of this note. For a sub-σ-finite algebra $T^{-1}(\Sigma) \subseteq \Sigma$, the conditional expectation operator associated with $T^{-1}(\Sigma)$ is the mapping $f \to E^{T^{-1}(\Sigma)}f$, defined for all non-negative f as well as for all $f \in L^p(\Sigma), 1 \leq p \leq \infty$, where $E^{T^{-1}(\Sigma)}f$, by Radon-Nikodym theorem, is the unique $T^{-1}(\Sigma)$-measurable function satisfying
$$\int_A f d\mu = \int_A E^{T^{-1}(\Sigma)} f d\mu, \quad \forall A \in T^{-1}(\Sigma).$$
Throughout this paper, we assume that $E^{T^{-1}(\Sigma)} = E$. For more details on the properties of the conditional expectation operators see [2, 4].

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