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On graded generalized local cohomology modules

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Abstract

Let M and N be two finitely generated graded modules over a standard graded Noetherian ring $R = \bigoplus_{n\geq 0} R_n$. In this paper we show that if R_0 is semi-local of dimension ≤ 2 then, the set $\mathrm{Ass}_{R_0}\Big(H^i_{R_+}(M,N)_n\Big)$ is asymptotically stable for $n \to -\infty$ in some special cases. Also, we study the torsion-freeness of graded generalized local cohomology modules $H^i_{R_+}(M,N)$. Finally, the tame loci $T^i(M,N)$ of (M,N) are introduced and some sufficient conditions are proposed for the openness of these sets in Zariski topology.

Keywords: generalized local cohomology modules, associated prime ideals, tame loci
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1 Introduction

Assume that R is a commutative Noetherian ring with identity and all modules are unitary. Let $\mathfrak{a}$ be an ideal of R and $R - Mod$ the category of R-modules and R-homomorphisms. We denote by $\mathbb{N}_0$ and $\mathbb{N}$ the sets of non-negative and positive integers, respectively.

For $i \in \mathbb{N}_0$, the i-th generalized local cohomology functor with respect to $\mathfrak{a}$ is a generalization of the i-th local cohomology functor with respect to $\mathfrak{a}$, i.e. $H^i_{\mathfrak{a}}(-) = \varinjlim_{n\in\mathbb{N}} \mathrm{Ext}^i_R(R/\mathfrak{a}^n, -)$([1], [5]). It is defined, by Herzog ([6]), as follows:

$$H^i_{\mathfrak{a}}(-,-) : R - Mod \times R - Mod \to R - Mod$$

$$H^i_{\mathfrak{a}}(M,N) = \varinjlim_{n\in\mathbb{N}} \mathrm{Ext}^i_R(M/\mathfrak{a}^n M, N).$$

For all R-modules M and N, $H^i_{\mathfrak{a}}(M,N)$ is called the i-th generalized local cohomology module of M and N with respect to $\mathfrak{a}$. These functors coincide when $M = R$ and have been studied by many authors (see for instance [2], [3].

Now, let $R = \bigoplus_{n\in\mathbb{N}_0} R_n$ be a standard graded Noetherian ring and let M and N be two finitely generated graded R-modules. Also, assume that $R_+ = \bigoplus_{n\in\mathbb{N}} R_n$ denotes the irrelevant ideal of R. It is well known that for each $i \in \mathbb{N}_0$, $H^i_{R_+}(M,N)$ carries a natural

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