Existence of Solution for G-BSDE with Quadratic Growth and Unbounded Terminal Value

Mojtaba Maleki
University of Shahrood

Elham Dastranj
University of Shahrood

Arazmohammad Arazi*
University of Shahrood

Abstract

In this paper, we present the existence of solutions for G-backward stochastic differential equations with quadratic growth and unbounded terminal value, under some assumptions.

Keywords: G-expectation, G-Brownian motion, G-Backward stochastic differential equations, quadratic growth, unbounded terminal value .
Mathematics Subject Classification [2010]: 13D45, 39B42

1 Introduction

We consider the following G-backward stochastic differential equation:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s dB_s - (K_T - K_t), \tag{1}$$

where K is a decreasing G-martingale. The terminal value ξ and the generator f are given. B_t is the G-Brownian motion. We present the existence of a solution (Y, Z, K) for (1) (see Theorems 3.1) in the G-framework.

2 Preliminaries

We briefly recall some basic notions of G-expectation. Let $(\Omega, \mathcal{H}, \mathbb{E})$ be the G-expectation space. We denote by $lip(\mathbb{R}^n)$ the space of all bounded and Lipschitz real functions on $\mathbb{R}^n$. In this paper we set $G(a) = \frac{1}{2}(a^+ - \sigma_0^2 a^-)$, where $a \in \mathbb{R}$ and $\sigma_0 \in [0, 1]$ is fixed.
Let $\Omega = \mathbb{R}$ and $\mathcal{H} = lip(\mathbb{R})$, in [1], X with G-normal distribution (with mean at $x \in \mathbb{R}$ and variance $t > 0$), is defined by

$$\mathbb{E}[\varphi(x + \sqrt{t}X)] = P_G^t(\varphi(x)) := u(t, x),$$

Where $\varphi \in lip(\mathbb{R})$ and $u = u(t, x)$ is a bounded continuous function on $[0, \infty) \times \mathbb{R}$ which is the solution of the following G-heat equation

$$\partial_t u - G(\partial_{xx}^2 u) = 0, \quad u(0, x) = \varphi(x).$$

*Speaker