# Existence and uniqueness of positive solutions for a class of singular nonlinear fractional differential equations with integral boundary value conditions

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**Abstract**

In this paper, we prove the existence and uniqueness of positive solutions for the following singular nonlinear fractional boundary value problem

$$^{c}D^{\alpha}u(t) + f(t, u(t), u(t), u(t)) = 0, \quad 0 < t < 1,$$

$$u(0) = u''(0) = u'''(0) = 0, \qquad u'(0) + u'(1) = \lambda \int_0^1 u(s)ds,$$

where $3 < \alpha \leq 4$, $0 < \lambda < 4$, $^{c}D^{\alpha}$ is the Caputo fractional derivative and $f : (0,1] \times [0,\infty) \times [0,\infty) \times [0,\infty) \to [0,\infty)$ is continuous, $\lim_{t\to 0^+} f(t,.,.,.) = +\infty$ ( i.e. $f$ is singular at $t = 0$). Our analysis is based on a tripled fixed point theorem in partially ordered metric spaces. An example is presented to illustrate the main results.

**Keywords:** Caputo fractional derivative, Positive solutions, Singular fractional equations, Fixed point
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## 1 Introduction

Fractional calculus is an extended concept of integral ones and fractional differential equations are widely used in various fields of sciences. There are some papers dealing with the existence of positive solutions for nonlinear fractional differential equations, see ([1], [2], [3], [4], [5]).

In this paper, we investigate the existence and uniqueness of positive solutions for the following singular nonlinear fractional boundary value problem

$$\begin{cases} ^{c}D^{\alpha}u(t) + f(t, u(t), u(t), u(t)) = 0, \quad 0 < t < 1, \\ u(0) = u''(0) = u'''(0) = 0, \qquad u'(0) + u'(1) = \lambda \displaystyle\int_0^1 u(s)ds, \end{cases} \tag{1}$$

where $3 < \alpha \leq 4$, $0 < \lambda < 4$, $^{c}D^{\alpha}$ is the Caputo fractional derivative and $f : (0,1] \times [0,\infty) \times [0,\infty) \times [0,\infty) \to [0,\infty)$ is continuous, $\lim_{t\to 0^+} f(t,.,.,.) = +\infty$ ( i.e. $f$ is singular at $t = 0$).

Our analysis is based on a new tripled fixed point theorem in partially ordered metric spaces.

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