Derivations of direct limits of Lie superalgebras

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Abstract

In this work, we study derivations of a direct limit of Lie superalgebras. As an application, we determine the derivation algebra of a direct union of finite dimensional basic classical simple Lie superalgebras.

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1 Derivations

Following the interest of physicists in the context of supersymmetries, in 1977, V. Kac [1] introduced Lie superalgebras (known as $\mathbb{Z}_2$-graded Lie algebras in Physics). He classified classical Lie superalgebras, i.e., finite dimensional simple Lie superalgebras whose even parts are reductive Lie algebras. These Lie superalgebras are a generalization of finite dimensional simple Lie algebras over an algebraically closed field of characteristic zero but classical Lie superalgebras are not necessarily equipped with nondegenerate invariant bilinear forms while Killing form on a finite dimensional simple Lie algebra over a field of characteristic zero is invariant and nondegenerate. To get a better super version of finite dimensional simple Lie algebras, one can work with those classical Lie superalgebras equipped with even nondegenerate invariant bilinear forms, called finite dimensional basic classical simple Lie superalgebras. It is known that all derivations of a finite dimensional Lie superalgebra with nondegenerate Killing form are inner. In [2], the author studies derivations of locally finite split simple Lie algebras [3]; a locally finite split simple Lie algebra is a direct union of finite dimensional split simple Lie algebras. In this work, we first study derivations of a direct limit of Lie superalgebras and then as an application, we determine the derivations of locally finite basic classical simple Lie superalgebras [4]. This work has been derived from the author's recent preprint on the topic.

Throughout this work, $\mathbb{F}$ is an algebraically closed field of characteristic zero. Unless otherwise mentioned, all vector spaces are considered over $\mathbb{F}$. We denote the dual space of a vector space V by V^*. We denote the degree of a homogenous element v of a superspace by $|v|$ and make a convention that if in an expression, we use $|u|$ for an element u of a superspace, by default we have assumed u is homogeneous. For two symbols i, j, by $\delta_{i,j}$, we mean the Kronecker delta.

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