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A generalization of α-dominating set and its complexity

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Abstract

Let $G = (V, E)$ be a simple and undirected graph. For some real number α with $0 < \alpha \leq 1$, a set $D \subseteq V$ is called an *α-dominating set* in G if every vertex v outside D has at least $\alpha \cdot d_v$ neighbor(s) in S where d_v is the degree of v. The cardinality of a minimum α-dominating set in a graph G is called the *α-domination number* of G and denoted by $\gamma_\alpha(G)$. In this paper, we introduce a generalization of α-dominating set, that we call it *f_{deg}-dominating set*. Given a function f_{deg} where f_{deg} is as $f_{deg} : \mathbb{N} \to \mathbb{R}$ where $\mathbb{N} = \{1, 2, 3, \ldots\}$, and f_{deg} may not be an integer-value function. A set $D \subseteq V$ is called an f_{deg}-dominating set in G if for every vertex v outside D, $|N(v) \cap D| \geq f_{deg}(d_v)$. In this paper, for this new concept, we will present some results on the its NP-completeness, APX-completeness and inapproximability.

Keywords: Domination, α-Domination, k-Domination, APX-Complete, NP-Complete
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1 Introduction

Let $G = (V, E)$ be an undirected and simple graph. A set $D \subseteq V$ is called a *dominating* set if every vertex outside D has at least one neighbor in D. The cardinality of a minimum dominating set is called the *domination number* of G denoted by $\gamma(G)$. In 2000, Dunbar et al. [5], introduced the concept of α-domination. Let α be a real number with $0 < \alpha \leq 1$. A set $D \subseteq V$ is called an α-dominating set in G if for every vertex v outside D, $|N(v) \cap D| \geq \alpha \times d_v$ where $N(v)$ is the set of all neighbors of v in G, and $d_v := |N(v)|$ is the degree of v. Also, let k be a real number with $k \geq 1$. A set $D \subseteq V$ is called a *k-dominating* set in G if for every vertex v outside D, $|N(v) \cap D| \geq k$.

Now consider the definition of α-dominating. One generalization of this concept is that instead of having at least $\alpha \times d_v$ neighbors in D for each vertex $v \notin D$, we have at least $f(d_v)$ neighbors in D, for some special function f. By selecting $f(x) = \alpha x$, the definition match the α-dominating. It seems that this generalization is much near to the reality. Hence, in this paper, we define the *f_{deg}-dominating* set. Given a function f_{deg} where f_{deg} is as $f_{deg} : \mathbb{N} \to \mathbb{R}$ where $\mathbb{N} = \{1, 2, 3, \ldots\}$, and f_{deg} may not be an integer-value function. A set $D \subseteq V$ is called an f_{deg}-dominating set in G if for every vertex v outside D, $|N(v) \cap D| \geq f_{deg}(d_v)$. In this paper, we consider the graphs with no isolated vertices. We can easily extend the results for the graphs with isolated vertices. In this

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