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Numerical Treatment of Coupling of Two Hyperbolic Conservation Laws By Local Discontinuous Galerkin Methods

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Abstract

In this work, the local discontinuous Galerkin (LDG) method is used to treat a system of differential equations consisting of two hyperbolic conservation laws. The cell entropy inequality is obtained when the upwind flux is utilized. In the linear case, we derive optimal convergence rates of order $\mathcal{O}(h^{k+1})$ in the L_2-norm, in domains where the exact solution is smooth; here h is the mesh width and k is the degree of the (orthogonal Legendre) polynomial functions spanning the finite element subspace. We justify the advantages of the LDG method in a series of numerical examples.

Keywords: Discontinuous Galerkin, coupling equations, error estimates
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1 Introduction

The main goal of this paper is to devise, analyze, and implement the local discontinuous Galerkin method (LDG) for the solution of the following coupling of two conservation laws in one space dimension: Find $u : (x,t) \in \mathbb{R} \times \mathbb{R}_+ \longrightarrow u(x,t) \in \mathbb{R}$ such that

$$\begin{cases} u_t + [f_R(u)]_x = 0, & x > 0, \quad t > 0, \\ u_t + [f_L(u)]_x = 0, & x < 0, \quad t > 0, \\ u(x,0) = u_0, & x \in \mathbb{R}, \end{cases} \tag{1}$$

and also a suitable "continuity" condition

$$u(x,t) = u^b(t) \quad t \geq 0,$$

at the interface $x = 0$, to be compatible with initial condition u_0, where $u_0 : \mathbb{R} \longrightarrow \mathbb{R}$ is a given function and $f_\alpha : \mathbb{R} \longrightarrow \mathbb{R}$, for $\alpha = L, R$, denote two "smooth" functions ([1, 3]). This type of phenomenon appears for example in an increasing number of problems of fluid mechanics, among others, we emphasize the case of coupled problems involving Euler equation on one side of the interface and Navier-Stokes equation on the other side, as well as modelling certain plasma physical problems cf [1].

For last decades, the technique of discontinuous Galerkin (DG) investigated as an higher-order accurate scheme for treating differential equations specially for those problems with hyperbolic nature and developing discontinuities [2]. The DG methods can

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